FINANCIJSKA MATEMATIKA

– FORMULE –

1. NIZOVI

Aritmetički niz

$$a_n = a_1 + (n-1)\cdot d\ , \qquad s_n = \frac{n}{2}\Big(2\cdot a_1 + (n-1)\cdot d\Big)\ , \qquad a_n = \frac{a_{n-1}+a_{n+1}}{2}$$

Geometrijski niz

$$a_n = a_1 \cdot q^{n-1}\ , \qquad s_n = a_1 \cdot \frac{q^n - 1}{q-1}\ , \qquad a_n = \sqrt{a_{n-1}\cdot a_{n+1}}$$

2. JEDNOSTAVNI *dekurzivni* KAMATNI RAČUN

$$I_n = C_0\ \frac{p\ n}{100}\ , \qquad C_n = C_0\Big(1 + \frac{p\ n}{100}\Big)$$

3. SLOŽENI *dekurzivni* KAMATNI RAČUN

$$I_n = C_{n-1}\frac{p}{100}\ , \qquad r = 1 + \frac{p}{100}\ , \qquad C_n = C_0 \cdot r^n$$

Ispodgodišnje ukamaćivanje

$$p_r = \frac{p}{m}\ , \qquad p' = 100\Big(\sqrt[m]{1+\frac{p}{100}} - 1\Big)\ , \qquad r' = \sqrt[m]{r}$$

Kontinuirano ukamaćivanje

$$C_n = C_0 \cdot e^{\frac{pn}{100}}$$

4. PERIODSKE UPLATE I ISPLATE

Konačna vrijednost periodskih uplata

$$S = Rr\frac{r^n - 1}{r-1}\ , \qquad S' = R\frac{r^n - 1}{r-1}$$

$$n = \frac{\log\Big(\frac{S(r-1)}{Rr} + 1\Big)}{\log r}\ , \qquad n = \frac{\log\Big(\frac{S'(r-1)}{R} + 1\Big)}{\log r}$$

Sadašnja vrijednost periodskih isplata

$$A = \frac{S}{r^n} = R\frac{r^n - 1}{r^{n-1}(r-1)} \ , \qquad A' = \frac{S'}{r^n} = R\frac{r^n - 1}{r^n(r-1)}$$

$$n = \frac{\log\left(\frac{Rr}{Rr - A(r-1)}\right)}{\log r} \ , \qquad n = \frac{\log\left(\frac{R}{R - A'(r-1)}\right)}{\log r}$$

$$R' = \left(A - R\frac{r^n - 1}{r^{n-1}(r-1)}\right) \cdot r^n \ , \qquad R' = \left(A' - R\frac{r^n - 1}{r^n(r-1)}\right) \cdot r^{n+1}$$

Beskonačna renta

$$A_\infty = \frac{R}{r-1}$$

Varijabilne periodske uplate i isplate

Sadašnja vrijednost isplata koje čine aritmetički niz

$$\hat{A} = \frac{Rr}{r-1}\left(\frac{r^n - 1}{r^{n-1}(r-1)} - \frac{n}{r^n}\right) \ , \quad \hat{A}' = \frac{R}{r-1}\left(\frac{r^n - 1}{r^{n-1}(r-1)} - \frac{n}{r^n}\right)$$

Buduća vrijednost uplata koje čine geometrijski niz

$$\hat{S} = Rr\frac{q^n - r^n}{q - r} \ , \qquad \hat{S}' = R\frac{q^n - r^n}{q - r}$$

5. KREDIT

Otplata kredita jednakim anuitetima krajem razdoblja

$$K = a\frac{r^n - 1}{r^n(r-1)} \ , \qquad a = K\frac{r^n(r-1)}{r^n - 1}$$

$$I_k = \frac{O_{k-1}p'}{100} \ , \quad I_k = O_{k-1}(r-1) \ , \quad R_k = a - I_k \ , \quad O_k = O_{k-1} - R_k$$

$$O_k = a\frac{r^{n-k} - 1}{r^{n-k}(r-1)} \ , \quad R_k = R_1 r^{k-1} \ , \quad R_k = \frac{a}{r^{n-k+1}}$$

Otplata kredita jednakim otplatnim kvotama

$$R = \frac{K}{n} \ , \quad I_k = O_{k-1}(r-1) \ , \quad a = R + I_k \ , \quad O_k = O_{k-1} - R$$

6. POKAZATELJI ISPLATIVOSTI ULAGANJA

Čista sadašnja vrijednost

$$NPV = F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \cdots + \frac{F_n}{r^n}$$

Interna stopa profitabilnosti

$$F_0 + \frac{F_1}{r} + \frac{F_2}{r^2} + \cdots + \frac{F_n}{r^n} = 0$$

7. AMORTIZACIJA

Linearna metoda

$$R = \frac{C - S}{n} \ , \quad D_k = k \cdot R \ , \quad B_k = C - D_k$$

Metoda konstantnog postotka

$$R_k = B_{k-1}\frac{d}{100} \ , \quad B_k = B_{k-1} - R_k \ , \quad D_k = C - B_k \ , \quad S = C\left(1 - \frac{d}{100}\right)^n$$

Metoda sume znamenaka

$$s = 1 + 2 + \cdots + n$$

$$R_1 = \frac{n}{s}(C - S) \ , \ldots , \ R_k = \frac{n - k + 1}{s}(C - S) \ , \ldots , \ R_n = \frac{1}{s}(C - S)$$

Funkcionalna metoda

$$Q = p_1 + p_2 + \cdots + p_n \ , \quad a = \frac{C - S}{Q} \ , \quad R_i = p_i \cdot a$$

8. ANTICIPATIVAN OBRAČUN KAMATA

Jednostavni kamatni račun

$$I_n = C_n\frac{q \cdot n}{100} \ , \qquad C_n = C_0\left(\frac{100}{100 - qn}\right)$$

Složeni kamatni račun

$$C_n = C_0\left(\frac{100}{100 - q}\right)^n \ , \qquad \rho = \frac{100}{100 - q} \ , \qquad C_n = C_0 \cdot \rho^n$$

Ispodgodišnje ukamaćivanje

$$q_r = \frac{q}{m} \ , \qquad q' = 100\left(1 - \sqrt[m]{1 - \frac{q}{100}}\right) \ , \qquad \rho' = \sqrt[m]{\rho}$$

Otplata kredita jednakim anuitetima

$$K = a\frac{\rho^n - 1}{\rho^{n-1}(\rho - 1)} \ , \qquad a = K\frac{\rho^{n-1}(\rho - 1)}{\rho^n - 1}$$

$$I_0 = \frac{Kq}{100}\ , \quad R_k = (a - I_0)\rho^k\ , \quad I_k = a - R_k\ , \quad O_k = O_{k-1} - R_k$$

9. MATEMATIKA OSIGURANJA

- **Jednogodišnje vjerojatnosti doživljenja i smrti**

$$p_x = 1 - q_x = \frac{l_{x+1}}{l_x} \qquad q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$$

- **n godišnje vjerojatnosti doživljenja i smrti**

$$_np_x = \frac{l_{x+n}}{l_x} \qquad _nq_x = 1 -_n p_x = \frac{l_x - l_{x+n}}{l_x}$$

- **Osobne rente**
  Neodgođena doživotna osobna renta

$$\ddot{a}_x = 1 + \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \ldots + \frac{D_\omega}{D_x} = \frac{N_x}{D_x}$$

  Neodgođena osobna renta trajanja n godina

$$\ddot{a}_{x:n\rceil} = 1 + \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \ldots + \frac{D_{x+n-1}}{D_x} = \frac{N_x - N_{x+n}}{D_x}$$

a) OSIGURANJE ZA SLUČAJ DOŽIVLJENJA

$$_nE_x = \frac{D_{x+n}}{D_x}$$

b) OSIGURANJE ZA SLUČAJ SMRTI

Neodgođeno doživotno osiguranje za slučaj smrti

$$A_x = \frac{C_x}{D_x} + \frac{C_{x+1}}{D_x} + \ldots + \frac{C_\omega}{D_x} = \frac{M_x}{D_x}$$

Neodgođeno osiguranje za slučaj smrti u trajanju n godina

$$_{|n}A_x = \frac{C_x}{D_x} + \frac{C_{x+1}}{D_x} + \ldots + \frac{C_{x+n-1}}{D_x} = \frac{M_x - M_{x+n}}{D_x}$$

c) MJEŠOVITO OSIGURANJE

$$A_{x:n\rceil} =_n E_x +_{|n} A_x = \frac{D_{x+n} + M_x - M_{x+n}}{D_x}$$

$$A^a_{x:n\rceil} = \frac{A_{x:n\rceil} + \alpha + \gamma \cdot \ddot{a}_{x:n\rceil}}{1 - \beta} \qquad P^a_{x:n\rceil} = \frac{A_{x:n\rceil} + \alpha + \gamma \cdot \ddot{a}_{x:n\rceil}}{(1 - \beta) \cdot \ddot{a}_{x:n\rceil}}$$