

Veselko Tomljenović

OSNOVE ELEKTROTEHNIKE 2

zbirka rješenja



Zagreb, 2008. godine

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Objavljivanje ovog nastavnog materijala odobrilo je
Stručno vijeće Tehničkog veleučilišta u Zagrebu
Zaključkom br. od .

PREDGOVOR

Izlaskom udžbenika Osnove Elektrotehnike II profesora dr.sc. Branislava Kuzmanovića, koji osim riješenih primjera sadrži i niz zadataka za vježbu, pokazala se potreba da te zadatke riješim. Naime, na konzultacije su mi dolazili studenti koji bi negdje u izradi zadataka zapeli i tražili bi pomoć. Da ne bih više puta rješavao uvijek iznova iste zadatke, učinilo mi se da je bolje da ih riješim sve. Takva rješenja u rukopisu sam dao kolegama asistentima da ih pogledaju i ukažu na eventualne greške. Međutim, interes studenata je bio tako velik da je ta prva verzija rješenja procurila i od tada se puno kopirala i koristila. U međuvremenu sam neka rješenja popravio i ispravio uočene greške, te sam ovu zbirku rješenja koristio na auditornim vježbama.

Mislim da će ova zbirka rješenja olakšati mnogim studentima svladavanje gradiva, jer je korisno imati uvid u to kako je moguće pristupiti pojedinim problemima. Ne tvrdim da su svi zadaci riješeni na najbolji mogući i najelegantniji način. Stoga pozivam studente da ove zadatke pokušaju riješiti i nekako drugačije, svojim vlastitim pristupom. Rado bih da mi takve primjere, kao i mjesta gdje mi se potkrala eventualna pogreška, pošalju elektroničkom poštom na adresu veselko.tomljenovic@tvz.hr.

Kad se ovi zadaci koriste na auditornim vježbama za ilustraciju gradiva, onda uz ovaj tekst treba još mnogo toga komentirati i objasniti, što nije moglo biti napisano u rješenjima radi preglednosti i sažetosti. Potrebno je npr. raspraviti neke druge mogućnosti rješavanja pojedinog zadatka. Stoga ovu zbirku rješenja ne bi bilo dobro shvatiti kao ispriku za nepohađanje auditornih vježbi.

Jako je važno da rad studenata ne zastane tek na proučavanju tuđih (i ovih mojih) rješenja. Neophodno je da se osposobe samostalno rješavati zadatke odgovarajuće težine i složenosti. U tome će im pomoći i druge zbirke zadataka koje preporučujemo u literaturi.

Treba istaći da su ovoj publikaciji svojim radom na predmetu doprinijeli i drugi sadašnji i prijašnji djelatnici ove ustanove svojim zadacima na pismenim ispitima.

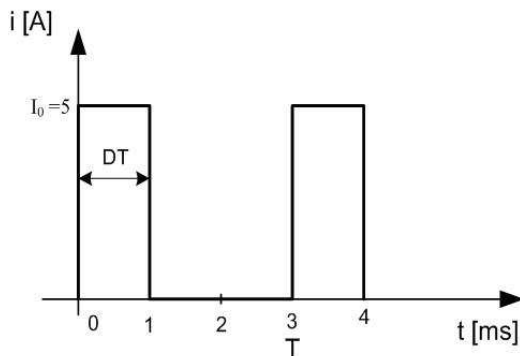
Ugodna mi je dužnost zahvaliti se studentima Nikoli Vodopijku i Matiji Cvetkoviću koji su vrlo uspješno pripremili slike i prepisali tekst.

Zagreb, 2008.

Veselko Tomljenović

1. Osnovni zakoni, elementi i parametri električnih krugova

1.1. Kroz otpor $R = 10 \, \Omega$ teče struja valnog oblika kao na slici. Odredite napon u , snagu p , radnu snagu P te efektivnu i srednju vrijednost struje.



a) $0 \leq t \leq 1 \, ms$

$$u = i \cdot R = 5 \cdot 10 = 50 \, V$$

$$p = u \cdot i = 50 \cdot 5 = 250 \, W$$

b) $1 \, ms \leq t \leq 3 \, ms$

$$u = 0$$

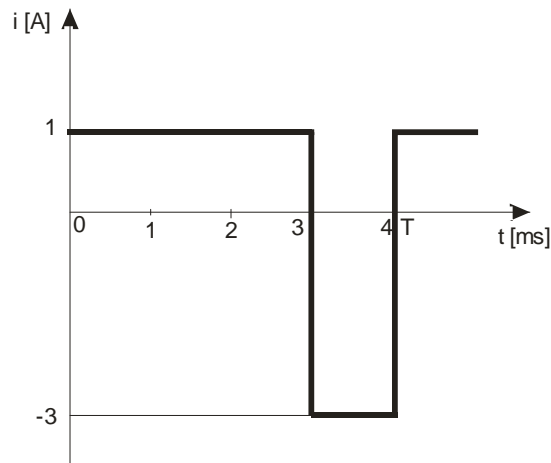
$$p = 0$$

$$I_{sr} = \frac{1}{T} \int_0^{DT} I_0 dt = DI_0 = \frac{1}{3} \cdot 5 = 1,67 \, A$$

$$I^2 = \frac{1}{T} \int_0^{DT} I_0^2 dt = DI_0^2 \rightarrow I = I_0 \sqrt{D} = 5 \sqrt{\frac{1}{3}} = 2,887 \, A$$

$$P = I^2 R = 2,887^2 \cdot 10 = 83,3 \, W$$

1.2. Kroz otpor $R = 10 \, \Omega$ teče struja valnog oblika kao na slici. Odredite napon u , snagu p , radnu snagu P te efektivnu i srednju “ispravljenu” vrijednost struje.



a)

$$0 < t < 3 \, ms$$

$$u = i \cdot R = 1 \cdot 10 = 10 \, V$$

$$p = i \cdot u = 1 \cdot 10 = 10 \, W$$

b)

$$3 \, ms < t < 4 \, ms$$

$$u = i \cdot R = -3 \cdot 10 = -30 \, V$$

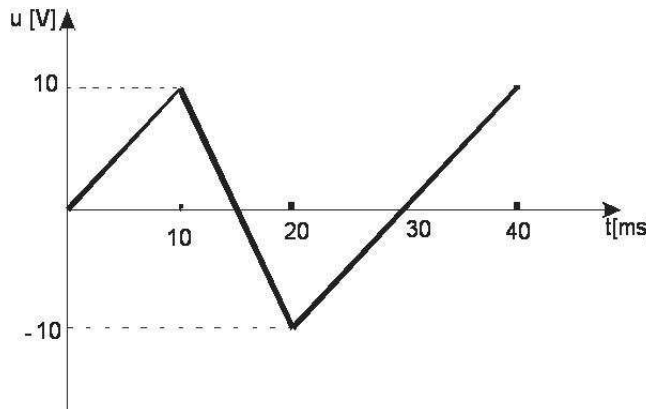
$$p = i \cdot u = -3 \cdot (-30) = 90 \, W$$

$$I_{sr} = \frac{1}{T} \int_0^T |i| dt = \frac{1}{4 \cdot 10^{-3}} (1 \cdot 10^{-3} + 3 \cdot 10^{-3}) = \frac{6}{4} = 1,5 \, A$$

$$I^2 = \frac{1}{T} \int_0^T i^2 dt = \frac{1}{4 \cdot 10^{-3}} (1^2 \cdot 3 \cdot 10^{-3} + 3^2 \cdot 10^{-3}) = \frac{12}{4} = 3 \rightarrow I = \sqrt{3} = 1,73 \, A$$

$$P = I^2 R = 3 \cdot 10 = 30 \, W$$

1.3. Otpor $R = 10 \Omega$ priključen je na napon valnog oblika kao na slici. Odredite struju $i(t)$, snagu p , srednju snagu P te efektivnu i srednju “ispravljenu” vrijednost struje.



a)

$$0 < t \leq 10 \text{ ms}$$

$$u(t) = \frac{10V}{10 \cdot 10^{-3} s} \cdot t = 10^3 t \text{ V}$$

$$i(t) = \frac{u}{R} = 100t \text{ A}$$

$$p(t) = u \cdot i = 10^3 t \cdot 100t = 10^5 t^2 \text{ W}$$

b)

$$10 \text{ ms} \leq t \leq 20 \text{ ms}$$

$$u(t) = -\frac{20V}{10 \cdot 10^{-3} s} t + 30 = -2000t + 30 \text{ V}$$

$$i(t) = \frac{u}{R} = -200t + 3 \text{ A}$$

$$p(t) = Ri^2 = 10(-200t + 3)^2 \text{ W}$$

c) $20 \text{ ms} \leq t \leq 30 \text{ ms}$

$$u(t) = 10^3 t - 30 \text{ V}$$

$$i(t) = \frac{u}{R} = 100t - 3 \text{ A}$$

$$p(t) = Ri^2 = 10(100t - 3)^2 \text{ W}$$

$$I_{sr} = \frac{1}{T} \int_0^T |i| dt = \frac{2}{30 \cdot 10^{-3}} \left(\frac{1 \cdot 10 \cdot 10^{-3}}{2} + \frac{1 \cdot 5 \cdot 10^{-3}}{2} \right) = \frac{1}{15} (5 + 2,5) = \frac{7,5}{15} = 0,5 \text{ A}$$

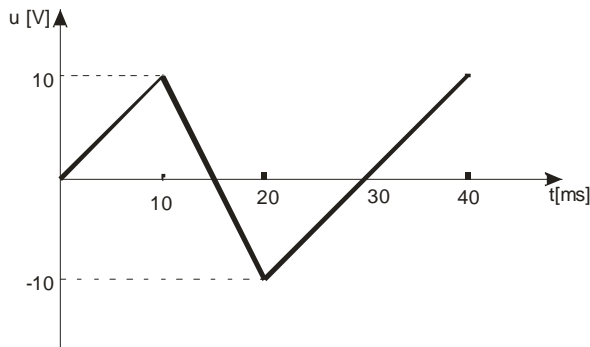
$$\begin{aligned} I^2 &= \frac{1}{T} \int_0^T i^2 dt = \frac{2}{30 \cdot 10^{-3}} \left[\int_0^{10^{-2}} 10^4 t^2 dt + \int_{10^{-2}}^{1,5 \cdot 10^{-2}} (4 \cdot 10^4 t^2 - 1200t + 9) dt \right] = \\ &= \frac{1}{15 \cdot 10^{-3}} \left[10^4 \frac{t^3}{3} \Big|_0^{10^{-2}} + \left(4 \cdot 10^4 \frac{t^3}{3} - 1200 \frac{t^2}{2} + 9t \right) \Big|_{10^{-2}}^{1,5 \cdot 10^{-2}} \right] \\ &= \frac{1}{15 \cdot 10^{-3}} \left[10^4 \cdot \frac{10^{-6}}{3} + \frac{4}{3} \cdot 10^4 \cdot 3,375 \cdot 10^{-6} - 600 \cdot 2,25 \cdot 10^{-4} + 9 \cdot 1,5 \cdot 10^{-2} - \frac{4}{3} \cdot 10^4 \cdot 10^{-6} + 600 \cdot 10^{-4} - 9 \cdot 10^{-2} \right] \\ &= \frac{1}{1,5 \cdot 10^{-2}} \left[\frac{1 \cdot 10^{-2}}{3} + \frac{13,5}{3} \cdot 10^{-2} - 13,5 \cdot 10^{-2} + 13,5 \cdot 10^{-2} - \frac{4}{3} \cdot 10^{-2} + 6 \cdot 10^{-2} - 9 \cdot 10^{-2} \right] \\ &= \frac{1}{1,5} \left(\frac{1}{3} + \frac{13,5}{3} - \frac{4}{3} + 6 - 9 \right) = \frac{1}{3} = 0,3 \\ I &= \sqrt{\frac{1}{3}} = 0,57735 \text{ A} \end{aligned}$$

Na drugi način:

$$I = \frac{I_m}{\sqrt{3}} = \frac{1}{\sqrt{3}} = 0,57735 \text{ A}$$

$$P = I^2 R = 0,33 \cdot 10 = 3,3 \text{ W}$$

1.4. Na kapacitet $C = 50 \mu\text{F}$ priključen je napon valnog oblika kao na slici. Odredite struju i , trenutnu snagu p i energiju w_C .



a)

$$0 < t < 10 \text{ ms}$$

$$u(t) = 10^3 t \text{ V}$$

$$i_C(t) = C \cdot \frac{d}{dt}(10^3 t)$$

$$i_C = 50 \cdot 10^{-6} \cdot 10^3 = 0,05 \text{ A}$$

$$p_C = ui_C = 10^3 t \cdot 0,05 = 50t \text{ VA}$$

$$w_C = q \frac{u}{2} = \frac{u^2 C}{2} = \frac{10^6 t^2 \cdot 50 \cdot 10^{-6}}{2} = 25t^2 \text{ Ws}$$

b)

$$10 \text{ ms} < t < 20 \text{ ms}$$

$$u(t) = -2000t + 30 \text{ V}$$

$$i_C = C \frac{d}{dt}(-2000t + 30) = 50 \cdot 10^{-6} \cdot (-2000) = -0,1 \text{ A}$$

$$p_C = ui_C = (-2000t + 30)(-0,1) = 200t - 3 \text{ VA}$$

$$w_C = \frac{u^2 C}{2} = \frac{(-2000 + 30)^2 \cdot 50 \cdot 10^{-6}}{2} = (4 \cdot 10^6 t^2 - 12 \cdot 10^4 t + 900) \cdot 2,5 \cdot 10^{-6}$$

$$= 100t^2 - 3t + 0,0225 \text{ Ws}$$

c)

$$20 \text{ ms} < t < 40 \text{ ms}$$

$$u(t) = 10^3 t - 30 \text{ V}$$

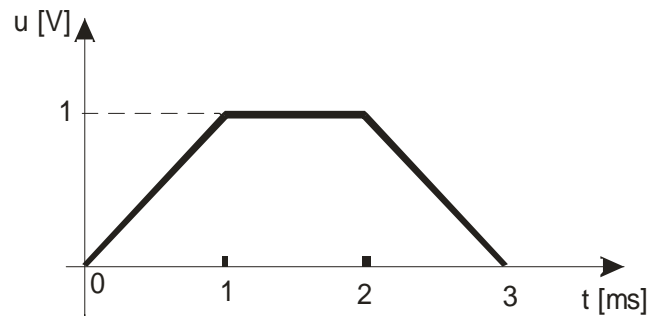
$$i_C = C \frac{d}{dt}(10^3 t - 30) = 50 \cdot 10^{-6} \cdot 10^3 = 0,05 \text{ A}$$

$$p_C = ui_C = (10^3 t - 30) \cdot 0,05 = 50t - 1,5 \text{ VA}$$

$$w_C = \frac{u^2 C}{2} = \frac{(10^3 t - 30)^2 \cdot 50 \cdot 10^{-6}}{2} = (10^6 t^2 - 6 \cdot 10^4 t + 900) \cdot 25 \cdot 10^{-6}$$

$$= 25t^2 - 1,5t + 0,0225 \text{ Ws}$$

1.5. Na kapacitet $C = 60 \mu\text{F}$ priključen je napon valnog oblika kao na slici. Odredite valni oblik struje i , trenutnu snagu p i energiju w_C .



a)

$$0 < t < 1 \text{ ms}$$

$$u(t) = \frac{1}{10^{-3}} t = 10^3 t \text{ V}$$

$$i_C = C \frac{d}{dt} u_C = 60 \cdot 10^{-6} \cdot 10^3 = 60 \text{ mA}$$

$$p_C = ui = 10^3 t \cdot 0,06 = 60t \text{ VA}$$

$$w_C = \frac{u^2 C}{2} = \frac{10^6 t^2 \cdot 60 \cdot 10^{-6}}{2} = 30t^2 \text{ Ws}$$

b)

$$1 \text{ ms} < t < 2 \text{ ms}$$

$$u = 1 \text{ V}$$

$$i_C = C \frac{du}{dt} = 0 \text{ A}$$

$$p_C = ui = 0 \text{ VA}$$

$$w_C = \frac{u^2 C}{2} = \frac{1 \cdot 60 \cdot 10^{-6}}{2} = 30 \cdot 10^{-6} \text{ Ws}$$

c)

$$2 \text{ ms} < t < 3 \text{ ms}$$

$$u(t) = -10^3 t + 3 \text{ V}$$

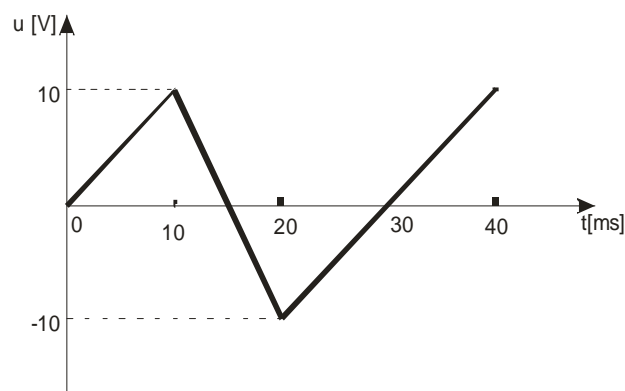
$$i_C = C \frac{d}{dt} u_C = 60 \cdot 10^{-6} \cdot (-10^3) = -60 \text{ mA}$$

$$p_C = ui = (-10^3 t + 3) \cdot (-0,06) = 60t - 0,18 \text{ VA}$$

$$w_C = \frac{u^2 C}{2} = \frac{(-10^3 t + 3)^2 \cdot 60 \cdot 10^{-6}}{2} = (10^6 t^2 - 6 \cdot 10^3 t + 9) \cdot 30 \cdot 10^{-6}$$

$$w_C = 30t^2 - 0,18t + 0,27 \cdot 10^{-3} \text{ Ws}$$

1.6. Na induktivitet $L = 1 \text{ mH}$ priključen je napon valnog oblika kao na slici. Odredite valni oblik struje i , trenutnu snagu p i energiju w_L .



a)

$$0 < t < 10 \text{ ms}$$

$$u(t) = 10^3 t \text{ V}$$

$$i_L = \frac{1}{L} \int u dt = 10^3 \int 10^3 t dt = \frac{10^6}{2} t^2 + 0 \text{ A}$$

$$i_L(10 \text{ ms}) = \frac{10^6}{2} \cdot 10^{-4} = 50 \text{ A}$$

$$p = u \cdot i_L = 10^3 t \cdot \frac{10^6}{2} t^2 = 0,5 \cdot 10^9 t^3 \text{ VA}$$

$$w_L = \frac{L \cdot i_L^2}{2} = \frac{10^{-3} \cdot 10^{12} t^4}{4 \cdot 2} = \frac{1}{8} \cdot 10^9 t^4 \text{ Ws}$$

b)

$$10 \text{ ms} < t < 20 \text{ ms}$$

$$u(t) = -2000t + 30 \text{ V}$$

$$i_L = 10^3 \int (-2000t + 30) dt + K = -2 \cdot 10^6 \frac{t^2}{2} + 3 \cdot 10^4 t + K$$

$$i_L(10 \text{ ms}) = 50 = -10^6 \cdot 10^{-4} + 3 \cdot 10^4 \cdot 10^{-2} + K \rightarrow K = -150$$

$$i_L = -10^6 t^2 + 3 \cdot 10^4 t - 150 \text{ A}$$

$$i_L(0,02) = -10^6 \cdot 0,02^2 + 3 \cdot 10^4 \cdot 0,02 - 150 = 50 \text{ A}$$

$$\begin{aligned} p(t) &= u \cdot i_L = (-2000t + 30)(-10^6 t^2 + 3 \cdot 10^4 t - 150) = \\ &= 2 \cdot 10^9 t^3 - 6 \cdot 10^7 t^2 + 3 \cdot 10^5 t - 3 \cdot 10^7 t^2 + 9 \cdot 10^5 t - 4500 = \\ &= 2 \cdot 10^9 t^3 - 9 \cdot 10^7 t^2 + 12 \cdot 10^5 t - 4500 \text{ VA} \end{aligned}$$

$$w_L = \frac{L \cdot i_L^2}{2} = \frac{10^{-3}}{2} (-10^6 t^2 + 3 \cdot 10^4 t - 150)^2 \text{ Ws}$$

$$c) \quad 20 \text{ ms} < t < 40 \text{ ms} : \quad u(t) = 10^3 t - 30 \text{ V}$$

$$i_L = 10^3 \int (10^3 t - 30) dt + K_1 = 10^6 \frac{t^2}{2} - 3 \cdot 10^4 t + K_1$$

$$i_L(0,02) = 10^6 \cdot 0,5 \cdot 0,02^2 - 3 \cdot 10^4 \cdot 0,02 + K_1 = 50 \text{ A} \rightarrow K_1 = 450$$

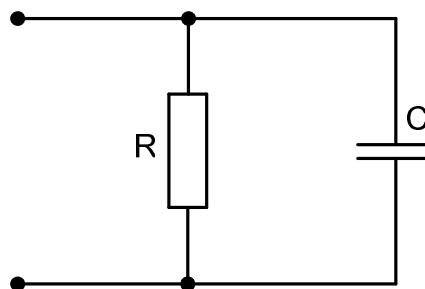
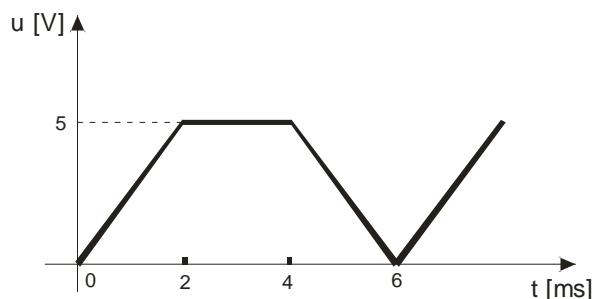
$$i_L = 5 \cdot 10^5 t^2 - 3 \cdot 10^4 t + 450 \text{ A}$$

$$i_L(0,03) = 0 \text{ A}$$

$$\begin{aligned} p(t) &= u \cdot i_L = (10^3 t - 30)(5 \cdot 10^5 t^2 - 3 \cdot 10^4 t + 450) = \\ &= 5 \cdot 10^8 t^3 - 3 \cdot 10^7 t^2 + 4,5 \cdot 10^5 t - 1,5 \cdot 10^7 t^2 + 9 \cdot 10^5 t - 13500 = \\ &= 5 \cdot 10^8 t^3 - 4,5 \cdot 10^7 t^2 + 13,5 \cdot 10^5 t - 13500 \text{ VA} \end{aligned}$$

$$w_L = \frac{L \cdot i_L^2}{2} = \frac{10^{-3}}{2} (5 \cdot 10^5 t^2 - 3 \cdot 10^4 t + 450)^2 \text{ J}$$

1.7. Paralelni spoj otpora $R = 5 \, \Omega$ i kapaciteta $C = 400 \, \mu\text{F}$ priključen je na napon valnog oblika kao na slici. Odredite sve struje (i_R , i_C , i), radnu snagu izvora P i maksimalnu energiju na kapacitetu C .



a)

$$0 < t \leq 2 \, \text{ms}$$

$$u = \frac{5}{2 \cdot 10^{-3} t} = 2500t \, \text{V}$$

$$i_R = \frac{u}{R} = \frac{2500t}{5} = 500t \, \text{A}$$

$$i_C = C \cdot \frac{du_C}{dt} = 400 \cdot 10^{-6} \cdot 2500 = 1 \, \text{A}$$

$$i = i_R + i_C = 500t + 1 \, \text{A}$$

b)

$$2 \, \text{ms} \leq t \leq 4 \, \text{ms}$$

$$u = 5 \, \text{V}$$

$$i_R = \frac{u}{R} = \frac{5}{5} = 1 \, \text{A}$$

$$i_C = 0$$

$$i = 1 \, \text{A}$$

c)

$$4 \, \text{ms} \leq t \leq 6 \, \text{ms}$$

$$u = -2500t + 15 \, \text{V}$$

$$i_R = \frac{u}{R} = \frac{-2500t + 15}{5} = -500t + 3 \, \text{A}$$

$$i_C = C \cdot \frac{du_C}{dt} = 400 \cdot 10^{-6} \cdot (-2500) = -1 \, \text{A}$$

$$i = -500t + 2 \, \text{A}$$

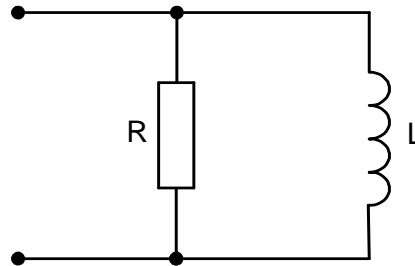
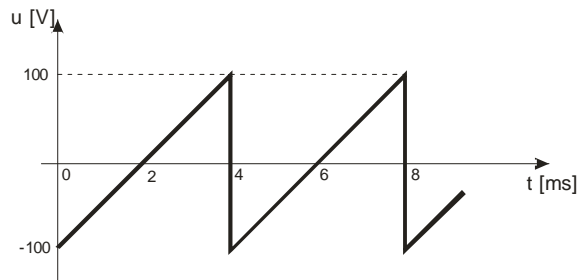
$$w_{C_{\max}} = \frac{u^2 C}{2} = \frac{25 \cdot 400 \cdot 10^{-6}}{2} = 5 \, \text{mJ}$$

$$I_R^2 = \frac{1}{T} \int_0^T i_R^2 dt = \frac{1}{T} \left[\int_0^{2 \, \text{ms}} (500t)^2 dt + \int_{2 \, \text{ms}}^{4 \, \text{ms}} 1^2 dt + \int_{4 \, \text{ms}}^{6 \, \text{ms}} (-500t + 3)^2 dt \right] =$$

$$= \frac{1}{T} \left[2 \cdot 500^2 \frac{t^3}{3} \Big|_0^{2 \, \text{ms}} + 0,002 \right] = \frac{1}{6 \cdot 10^{-3}} \left[0,001\dot{3} + 0,002 \right] = 0,5\dot{5} \, \text{A}^2$$

$$P = I_R^2 R = 0,5\dot{5} \cdot 5 = 2,7 \, \text{W}$$

1.8. Na paralelni spoj otpora $R = 10 \Omega$ i induktiviteta $L = 10 \text{ mH}$ priključen je napon valnog oblika kao na slici. Odredite sve struje (i_R , i_L , i), radnu snagu izvora P i maksimalnu energiju na induktivitetu L .



$$0 \leq t < 4 \text{ ms}$$

$$u = \frac{100t}{2 \cdot 10^{-3}} - 100 = 5 \cdot 10^4 t - 100 \text{ V}$$

$$i_R = \frac{u}{R} = 5 \cdot 10^3 t - 10 \text{ A}$$

$$i_L = \frac{1}{L} \int u dt = \frac{1}{0,01} \int (5 \cdot 10^4 t - 100) dt = 100 \left[5 \cdot 10^4 \frac{t^2}{2} - 100t \right] = 2,5 \cdot 10^6 t^2 - 10^4 t$$

$$i_L(4\text{ms}) = 2,5 \cdot 10^6 \cdot 4^2 \cdot 10^{-6} - 10^4 \cdot 4 \cdot 10^{-3} = 40 - 40 = 0 \text{ A}$$

$$i = i_R + i_L = 2,5 \cdot 10^6 t^2 - 5 \cdot 10^3 t - 10 \text{ A}$$

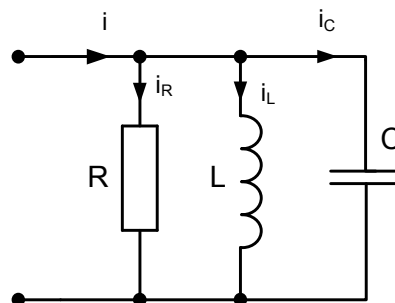
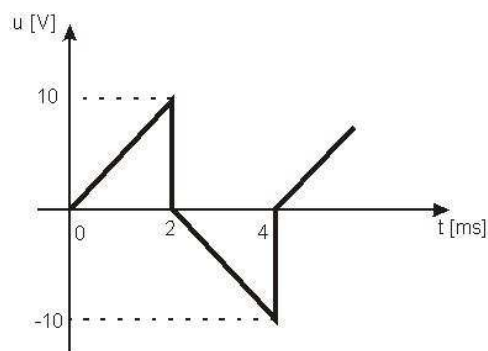
$$U^2 = \frac{1}{T} \int_0^T \frac{U_{\max}^2}{T^2} t^2 dt = \frac{U_{\max}^2}{T^3} \cdot \frac{t^3}{3} \Big|_0^T = \frac{U_{\max}^2}{3} \rightarrow U = \frac{U_{\max}}{\sqrt{3}} = \frac{100}{\sqrt{3}} = 57,735 \text{ V}$$

$$P = \frac{U^2}{R} = \frac{57,735^2}{10} = 333,3 \text{ W}$$

$$i_{L_{\max}}(2\text{ms}) = 2,5 \cdot 10^6 \cdot 2^2 \cdot 10^{-6} - 10^4 \cdot 2 \cdot 10^{-3} = 10 - 20 = -10 \text{ A}$$

$$w_{L_{\max}} = \frac{i_{L_{\max}}^2 \cdot L}{2} = \frac{10^2 \cdot 0,01}{2} = 0,5 \text{ J}$$

1.9. Na paralelni RLC spoj priključen je napon valnog oblika kao na slici. Zadano: $R = 2 \Omega$, $L = 1 \text{ mH}$, $C = 1 \text{ mF}$ i početni napon $u_C(0) = 0$. Odredite sve struje (i_R , i_L , i_C , i).



a)

$$0 \leq t < 2 \text{ ms}$$

$$u(t) = \frac{10}{2 \cdot 10^{-3}} t = 5000t \text{ V}$$

$$i_R = \frac{u}{R} = \frac{5000t}{2} = 2500t \text{ A}$$

$$i_C = C \frac{du_C}{dt} = 10^{-3} \cdot 5000 = 5 \text{ A}$$

$$i_L = \frac{1}{L} \int u dt = \frac{1}{10^{-3}} \int 5000t dt = 5 \cdot 10^6 \frac{t^2}{2} \text{ A}$$

$$i_L(2\text{ms}) = 2,5 \cdot 10^6 \cdot 4 \cdot 10^{-6} = 10 \text{ A}$$

$$i = i_R + i_C + i_L = 2,5 \cdot 10^6 t^2 + 2,5 \cdot 10^3 t + 5 \text{ A}$$

b)

$$2\text{ms} < t < 4 \text{ ms}$$

$$u(t) = -5000t + 10 \text{ V}$$

$$i_R = \frac{u}{R} = -2,5 \cdot 10^3 t + 5 \text{ A}$$

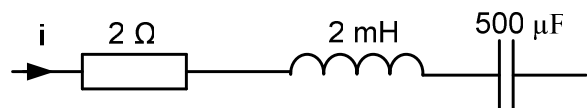
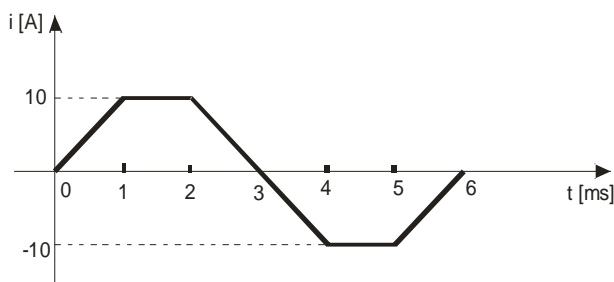
$$i_C = 10^{-3}(-5000) = -5 \text{ A}$$

$$i_L = \frac{1}{L} \int u dt + K = 10^3 \int (-5000t + 10) dt + K = 10^3 \left[-5000 \frac{t^2}{2} + 10t \right] + K = -2,5 \cdot 10^6 t^2 + 10^4 t + K$$

$$i_L(2\text{ms}) = 10 = -2,5 \cdot 10^6 \cdot 4 \cdot 10^{-6} + 10^4 \cdot 2 \cdot 10^{-3} + K = -10 + 20 + K \Rightarrow K = 0$$

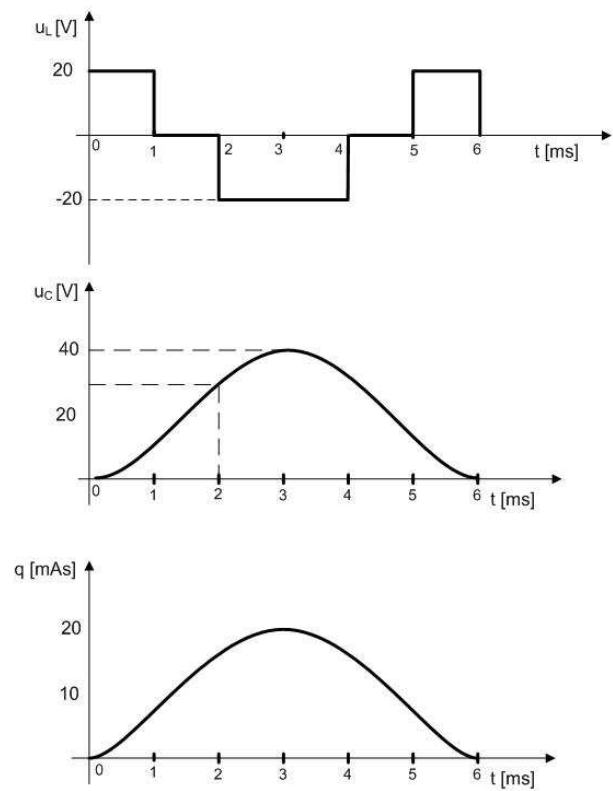
$$i = i_L + i_R + i_C = -2,5 \cdot 10^6 t^2 + 10^4 t - 2,5 \cdot 10^3 t + 5 - 5 = -2,5 \cdot 10^6 t^2 + 7,5 \cdot 10^3 t \text{ A}$$

1.10. Kroz serijski RLC spoj ($R = 2 \Omega$, $L = 2 \text{ mH}$, $C = 500 \mu\text{F}$, $u_C(0) = 0$) teče struja valnog oblika kao na slici. Odredite napone u_R , u_L i u_C .



Za određivanje napona primijenite relacije : $u_R = iR$, $u_L = L \frac{di}{dt}$ i $u_C = \frac{1}{C} \int i dt$

Naboj je $q = u_C \cdot C$; uz pozitivnu struju kondenzator se nabija (q raste), a uz negativnu se izbijla. Dijagrami su prikazani na slici.



a) $0 \leq t < 1 \text{ ms}$

$$i(t) = 10^4 t, A$$

$$u_R = i \cdot R = 20 \cdot 10^3 t, V$$

$$u_L = L \frac{di}{dt} = 2 \cdot 10^{-3} \cdot 10^4 = 20 V$$

$$u_C = \frac{1}{C} \int i dt =$$

$$= \frac{1}{500 \cdot 10^{-6}} \int 10^4 t dt = 20 \cdot 10^6 \cdot \frac{t^2}{2} = 10^7 t^2 V$$

$$u_C(10^{-3}) = 10 V$$

b) $1 \text{ ms} < t < 2 \text{ ms}$

$$i(t) = 10 A$$

$$u_R = i \cdot R = 20 V$$

$$u_L = 0 V$$

$$u_C = \frac{1}{C} \int i dt + K_1 = \frac{1}{500 \cdot 10^{-6}} \int 10 dt + K_1$$

$$= 2 \cdot 10^4 t + K_1$$

$$u_C(1\text{ ms}) = 10 = 2 \cdot 10^4 \cdot 10^{-3} + K_1 = 20 + K_1$$

$$\rightarrow K_1 = -10$$

$$u_C = 2 \cdot 10^4 t - 10\text{ V}$$

$$u_C(2\text{ ms}) = 30\text{ V}$$

$$\text{c) } 2\text{ ms} < t < 4\text{ ms}$$

$$i = -10^4 t + 30\text{ A}$$

$$u_R = -20 \cdot 10^{-3} t + 60\text{ V}$$

$$u_L = L \frac{di}{dt} = 2 \cdot 10^{-3} \cdot (-10^4) = -20\text{ V}$$

$$u_C = \frac{1}{C} \int i dt + K_2 = \frac{1}{500 \cdot 10^{-6}} \int (-10^4 t + 30) dt + K_2$$

$$= 2000 \left(-10^4 \cdot \frac{t^2}{2} + 30t \right) + K_2 = -10^7 t^2 + 6 \cdot 10^4 t + K_2$$

$$u_C(2\text{ ms}) = 30 = -10^7 \cdot 4 \cdot 10^{-6} + 6 \cdot 10^4 \cdot 2 \cdot 10^{-3} + K_2 \rightarrow K_2 = -50$$

$$u_C = -10^7 t^2 + 6 \cdot 10^4 t - 50\text{ V}$$

$$u_C(4\text{ ms}) = 30\text{ V}$$

$$\text{d) } 4\text{ ms} < t < 5\text{ ms}$$

$$i = -10\text{ A} \rightarrow u_L = 0\text{ V}$$

$$u_R = -20\text{ V}$$

$$u_C = \frac{1}{C} \int i dt + K_3 = \frac{1}{500 \cdot 10^{-6}} \int (-10) dt + K_3 = -2 \cdot 10^4 t + K_3$$

$$u_C(4\text{ ms}) = 30 = -2 \cdot 10^4 \cdot 4 \cdot 10^{-3} + K_3 \rightarrow K_3 = 110$$

$$u_C = -2 \cdot 10^4 t + 110\text{ V}$$

$$u_C(5\text{ ms}) = 10\text{ V}$$

$$\text{e) } 5\text{ ms} < t < 6\text{ ms}$$

$$i = 10^4 t - 60\text{ A}$$

$$u_R = 2 \cdot 10^4 t - 120\text{ V}$$

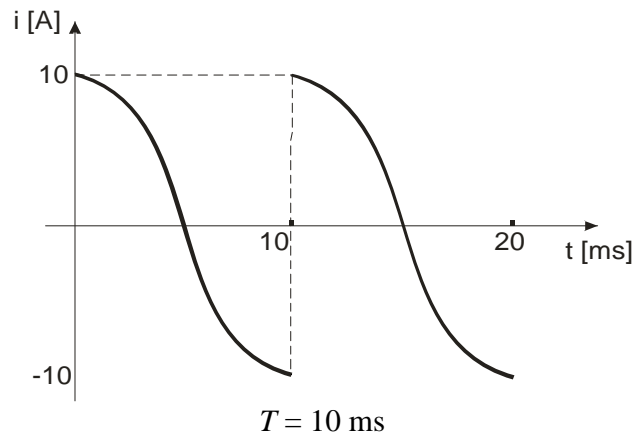
$$u_L = L \cdot \frac{di}{dt} = 20\text{ V}$$

$$u_C = \frac{1}{500 \cdot 10^{-6}} \int (10^4 t - 60) dt + K_4 = 2000 \left(\frac{10^4 t^2}{2} - 60t \right) + K_4 = 10^7 t^2 - 12 \cdot 10^4 t + K_4$$

$$u_C(5\text{ ms}) = 10 = 10^7 \cdot 25 \cdot 10^{-6} - 12 \cdot 10^4 \cdot 5 \cdot 10^{-3} + K_4 \rightarrow K_4 = 360$$

$$u_C = 10^7 t^2 - 12 \cdot 10^4 t + 360\text{ V}$$

1.11. Kroz serijski RC spoj teče struja valnog oblika kao na slici. Ako je $R = 10 \Omega$, $C = 0.1 \text{ mF}$ i $u_C(0) = 0$, odredite napone u_R i u_C , radnu snagu izvora P i efektivnu vrijednost struje.



a) $0 \leq t < 10 \text{ ms}$

$$\omega = \frac{2\pi}{2T} = \frac{\pi}{10^{-2}} = 10^2 \pi \text{ rad/s}$$

$$i = 10 \cos \omega t = 10 \cos(100\pi t) \text{ A}$$

$$u_R = R \cdot i = 100 \cos 314t \text{ V}$$

$$u_C = \frac{1}{C} \int i(t) dt + u_C(0)$$

$$= \frac{1}{10^{-4}} \int 10 \cos 314t dt$$

$$= 10^5 \cdot \frac{\sin 314t}{314} = 318,3 \sin 314t \text{ V}$$

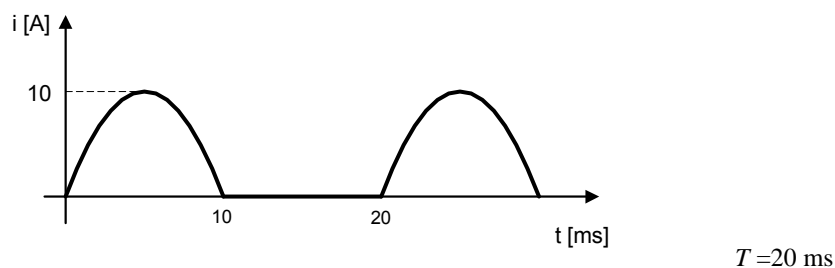
$$I = \frac{10}{\sqrt{2}} = 7,07 \text{ A} \rightarrow P = I^2 R = \frac{100}{2} \cdot 10 = 500 \text{ W}$$

b) $10 \text{ ms} < t < 20 \text{ ms}$

$$i = 10 \cos 314(t - 10^{-2}) \text{ A} \rightarrow u_R = 100 \cos 314(t - 10^{-2}) \text{ V}$$

$$u_C = 318,3 \sin 314(t - 10^{-2}) \text{ V}$$

1.12. Kroz serijski RL spoj teče struja valnog oblika kao na slici. Ako je $R = 10 \Omega$ i $L = 4 \text{ mH}$, odredite napone u_R i u_L , radnu snagu izvora P , efektivnu i srednju vrijednost struje, te faktor oblika.



$$a) 0 \leq t < 10 \text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{20 \cdot 10^{-3}} = 314 \frac{\text{rad}}{\text{s}}$$

$$i = 10 \sin \omega t = 10 \sin 314t \text{ A}$$

$$u_R = i \cdot R = 100 \sin 314t \text{ V}$$

$$u_L = L \cdot \frac{di}{dt} = 4 \cdot 10^{-3} \cdot 10 \cdot 314 \cos 314t$$

$$= 12,57 \cos 314t \text{ V}$$

$$b) 10 \leq t \leq 20 \text{ ms}$$

$$i_R = 0 \text{ A}; \quad u_R = 0 \text{ V}; \quad u_L = 0 \text{ V}$$

$$I_{sr} = \frac{1}{T} \int_0^{T/2} I_m \sin \omega t dt = \frac{\omega}{2\pi} \cdot \frac{I_m}{\omega} \cdot (-1) \cos \omega t \Big|_0^{T/2} = \frac{I_m}{2\pi} \cdot (-1) \cdot [-1 - 1] = \frac{I_m}{\pi} = \frac{10}{3} = 3,183 \text{ A}$$

$$I^2 = \frac{1}{T} \int_0^{T/2} I_m^2 \sin^2 \omega t dt = \frac{I_m^2}{T} \int_0^{T/2} \frac{1 - \cos 2\omega t}{2} dt = \frac{I_m^2}{T} \int_0^{T/2} \frac{dt}{2} - \frac{I_m^2}{T} \int_0^{T/2} \frac{\cos 2\omega t}{2} dt$$

$$I^2 = \frac{I_m^2}{4} \rightarrow I = \frac{I_m}{2} = \frac{10}{2} = 5 \text{ A}$$

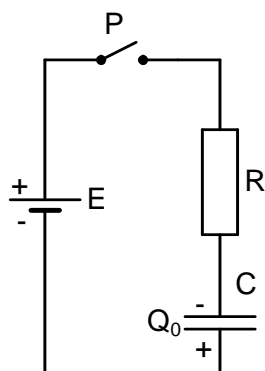
$$P = I^2 R = 2,5 \cdot 10 = 250 \text{ W}$$

$$\text{Faktor oblika } \xi = \frac{I}{I_{sr}} = \frac{5\pi}{10} = \frac{\pi}{2} = 1,571$$

$$\text{Na drugi na\u010din: } I = \frac{I_m}{\sqrt{2}} \sqrt{\frac{T_1}{T}} = \frac{10}{\sqrt{2}} \sqrt{\frac{10}{20}} = \frac{10}{\sqrt{2} \cdot \sqrt{2}} = 5 \text{ A}$$

2. Rješavanje mreža u vremenskoj domeni

2.1. Serijski spoj otpora $R = 1000 \, \Omega$ i kapaciteta $C = 10 \, \mu\text{F}$ priključuje se u trenutku $t = 0$ na istosmjerni naponski izvor $E = 100 \, \text{V}$. Ako je kapacitet prethodno posjedovao naboj $Q_0 = 1000 \, \mu\text{C}$, polariteta prema slici, treba odrediti struju i , napon na kapacitetu u_C i otporu u_R .



$$U_0 = \frac{Q_0}{C} = \frac{10^{-3}}{10^{-5}} = 100 \, \text{V}$$

$$I_0 = \frac{E + U_0}{R} = \frac{100 + 100}{1000} = 0,2 \, \text{A}$$

$$\tau = R \cdot C = 10^3 \cdot 10^{-5} = 10^{-2} = 0,01 \, \text{s}$$

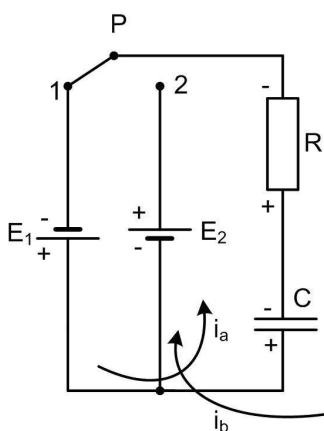
$$i = I_0 \cdot e^{-\frac{t}{\tau}} = 0,2 \cdot e^{-100t} \, \text{A}$$

$$u_R = i \cdot R = 200 \cdot e^{-100t} \, \text{V}$$

$$u_C = E - u_R = 100 - 200 \cdot e^{-100t} \, \text{V}$$

2.2. Serijski spoj otpora $R = 1000 \, \Omega$ i kapaciteta $C = 10 \, \mu\text{F}$ priključuje se u trenutku $t = 0$ na izvor $E_1 = 100 \, \text{V}$, a zatim se nakon jedne vremenske konstante kruga $t_1 = \tau$ sklopka prebaci u položaj 2 na izvor $E_2 = 200 \, \text{V}$, prema slici. Treba odrediti struju i te napone u_R i u_C .

$$\tau = RC = 10^3 \cdot 10^{-5} = 10^{-2} = 0,01 \, \text{s}$$



a) $0 \leq t < \tau$

$$I_{0a} = \frac{E_1}{R} = \frac{100}{1000} = 0,1 \, \text{A}$$

$$i_a = 0,1 \cdot e^{-100t} \, \text{A}$$

$$u_R = i_a \cdot R = 100 \cdot e^{-100t} \, \text{V}$$

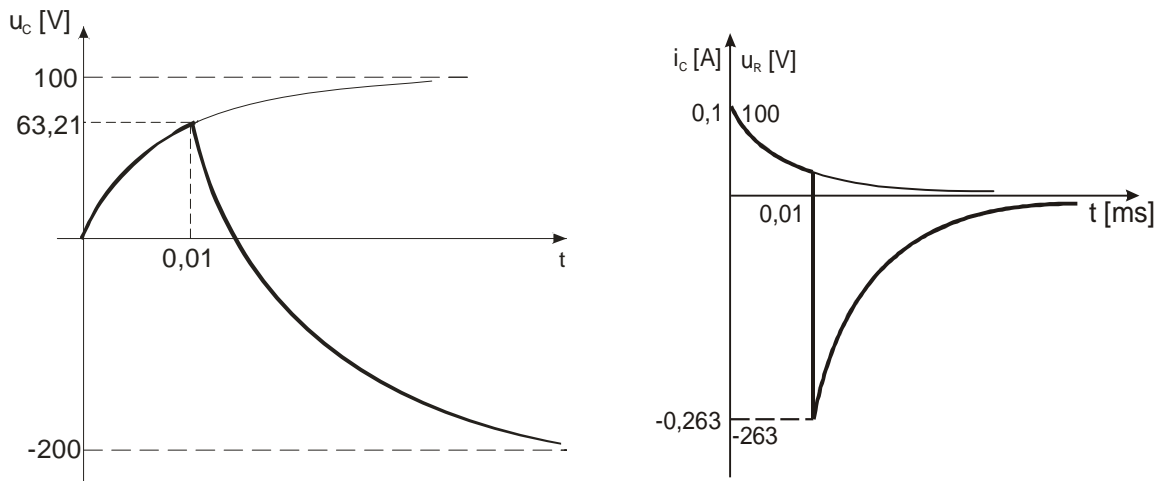
$$u_C = E_1 - u_R = 100 - 100 \cdot e^{-100t} = 100(1 - e^{-100t}) \, \text{V}$$

$$u_C(\tau) = 100(1 - e^{-1}) = 63,212 \, \text{V}$$

b) $\tau < t$
$$I_{0b} = -\left(\frac{u_C(\tau) + E_2}{R}\right) = -\frac{63,212 + 200}{1000} = -0,263 \, \text{A} \rightarrow i_b = -0,263 \cdot e^{-100(t-\tau)} \, \text{A}$$

$$u_R = i_b \cdot R = -263,2 \cdot e^{-100(t-\tau)} \, \text{V}$$

$$u_C + u_R = -E_2 \rightarrow u_C = -E_2 - u_R = -200 + 263,2 \cdot e^{-100(t-\tau)}, \, \text{V}$$



2.3. U zadatku 2.2. odredite utrošenu energiju izvora 1 i izvora 2 za vrijeme prijelazne pojave.

a) $0 \leq t < \tau$

$$w_a = \int_0^{\tau} E_1 i_a dt = \int_0^{\tau} 100 \cdot 0,1 \cdot e^{-100t} dt = \frac{10}{-100} \cdot e^{-100t} \Big|_0^{0,01} =$$

$$w_a = -0,1(e^{-100 \cdot 0,01} - e^0) = -0,1(0,3678 - 1) = 0,0632 \text{ J} = E_1 \cdot q_1 = 100 \cdot C \cdot u_C(\tau)$$

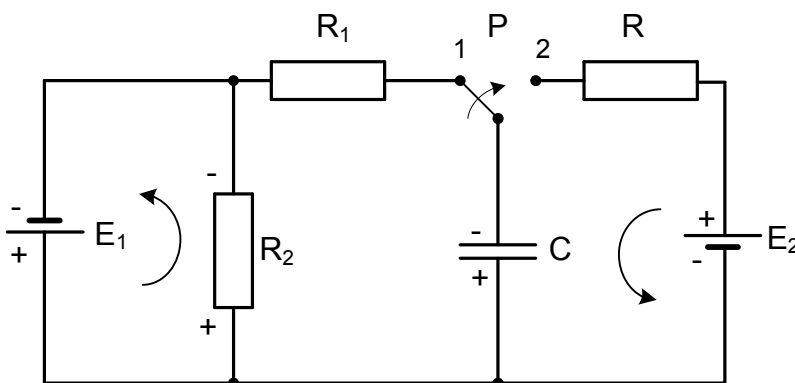
b) $\tau < t$

$$w_b = \int_{\tau}^{\infty} E_2 i_b dt = \int_{\tau}^{\infty} 200 \cdot 0,2632 \cdot e^{-100(t-\tau)} dt = \left. \begin{array}{l} u = t - \tau \\ du = dt \\ \text{za } t = \tau \rightarrow u = 0 \\ \text{za } t = \infty \rightarrow u = \infty \end{array} \right|$$

$$w_b = 52,64 \int_0^{\infty} e^{-100u} du = \frac{52,64}{-100} \cdot e^{-100u} \Big|_0^{\infty} = -0,5264(e^{-\infty} - e^0) = -0,5264(0 - 1) = 0,5264 \text{ J}$$

$$= E_2 \cdot q_2 = E_2[u_C(\tau) + E_2] \cdot C$$

2.4. U mreži prema slici prekidač P prebacuje se iz položaja 1 u položaj 2 u trenutku $t = 0$. Treba odrediti i_C i u_C . Zadano je: $E_1 = 150 \text{ V}$, $E_2 = 50 \text{ V}$, $R_1 = 5 \Omega$, $R_2 = 10 \Omega$, $R = 100 \Omega$, $C = 100 \mu\text{F}$.



$$u_C(0) = E_1 = 150 \text{ V} = U_0$$

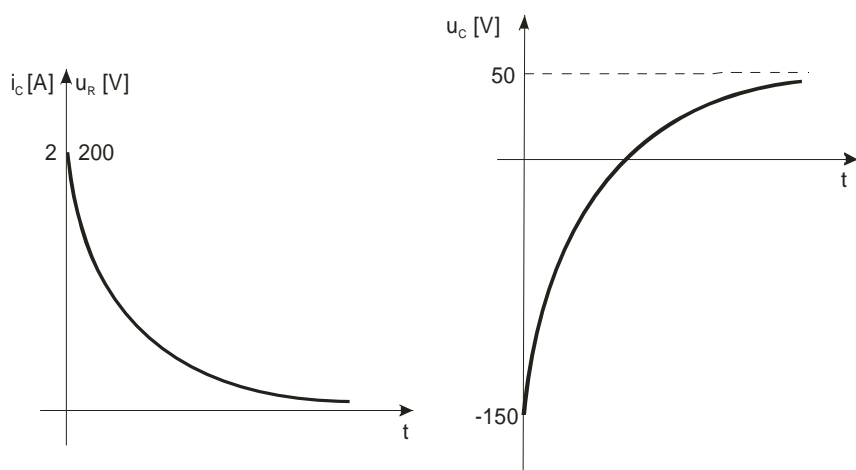
$$I_0 = \frac{E_2 + U_0}{R} = \frac{50 + 150}{100} = 2 \text{ A}$$

$$\tau = RC = 100 \cdot 100 \cdot 10^{-6} = 0,01 \text{ s}$$

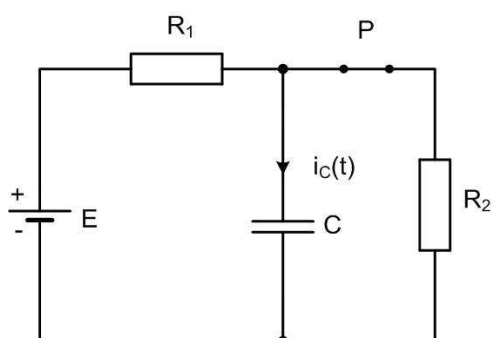
$$i_C = I_0 \cdot e^{\frac{-t}{\tau}} = 2e^{-100t} \text{ A}$$

$$u_R = i_C \cdot R = 200e^{-100t} \text{ V}$$

$$u_C = E_2 - u_R = 50 - 200e^{-100t} \text{ V}$$



2.5. Treba odrediti u_C , i_C i nacrtati njihove grafove nakon isključivanja prekidača P , prema slici. Zadano je: $E = 100 \text{ V}$, $R_1 = 30 \Omega$, $R_2 = 20 \Omega$, $C = 50 \mu\text{F}$.

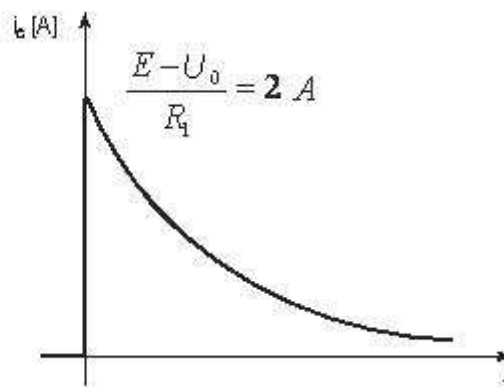
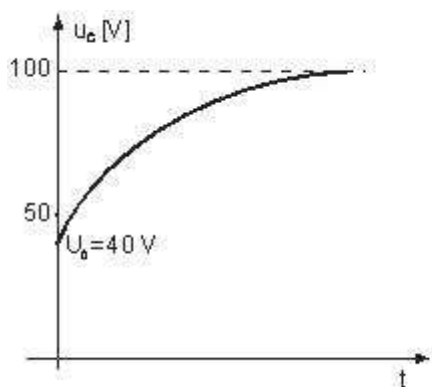


$$U_0 = E \cdot \frac{R_2}{R_1 + R_2} = 100 \cdot \frac{20}{30 + 20} = 40 \text{ V}$$

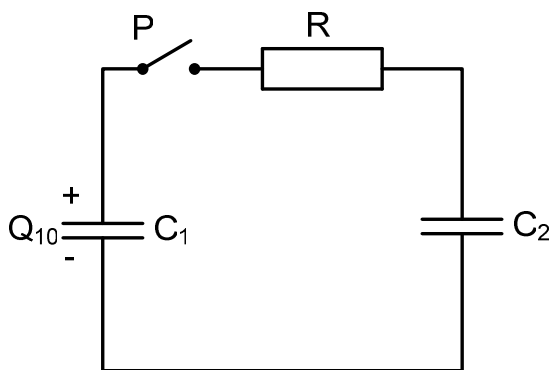
$$\tau = R_1 C = 30 \cdot 50 \cdot 10^{-6} = 1,5 \text{ ms}$$

$$u_C = U_0 + (E - U_0) \left(1 - e^{-\frac{t}{\tau}} \right) = 40 + 60 \cdot \left(1 - e^{-666,6t} \right) = 100 - 60 \cdot e^{-666,6t} \text{ V}$$

$$i_C = \frac{E - U_0}{R_1} \cdot e^{-\frac{t}{\tau}} = \frac{100 - 40}{30} \cdot e^{-666,6t} = 2 \cdot e^{-666,6t} \text{ A}$$



2.6. Treba odrediti struju i koja poteče od trenutka uključenja prekidača P i izračunati ukupne gubitke koji nastaju u strujnom krugu prema slici. Kapacitet $C_1 = 10 \text{ mF}$ nabijen je nabojem $Q_{10} = 10 \text{ mC}$, $C_2 = 20 \mu\text{F}$ nije nabijen, a $R = 10 \Omega$.



$$U_{10} = \frac{Q_{10}}{C_1} = \frac{10 \cdot 10^{-3}}{10 \cdot 10^{-3}} = 1 \text{ V}$$

$$C = \frac{C_1 \cdot C_2}{C_1 + C_2} = 19,96 \text{ } \mu\text{F}$$

$$\tau = RC = 10 \cdot 19,96 \cdot 10^{-6} = \frac{1}{5010} \text{ s}$$

$$I_0 = \frac{U_{10}}{R} = \frac{1}{10} = 0,1 \text{ A}$$

$$i = 0,1 \cdot e^{-5010t} \text{ , A}$$

$$W_{10} = \frac{C_1 \cdot U_{10}^2}{2} = \frac{10 \cdot 10^{-3} \cdot 1^2}{2} = 5 \text{ mJ}$$

$$U_{12} = \frac{Q_{10}}{C_{uk}} = \frac{10 \cdot 10^{-3}}{10 \cdot 10^{-3} + 20 \cdot 10^{-6}} = 0,998 \text{ V}$$

$$W_1 = \frac{C_1 U_{12}^2}{2} = \frac{10 \cdot 10^{-3} \cdot 0,998^2}{2} = 4,98 \text{ mJ}$$

$$W_2 = \frac{C_2 U_{12}^2}{2} = \frac{20 \cdot 10^{-6} \cdot 0,998^2}{2} = 9,96 \text{ J}$$

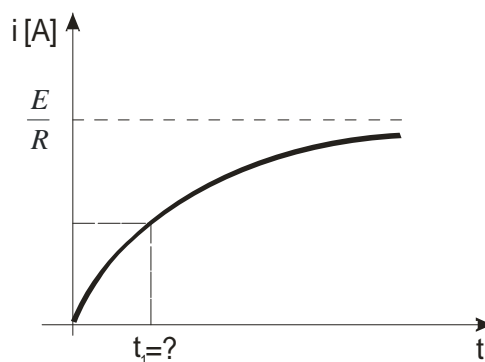
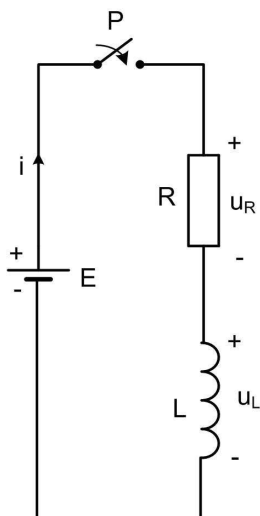
$$\Delta W = W_{10} - W_1 - W_2 = 9,98 \text{ } \mu\text{J}$$

$$\text{ili } W_R = \int_0^\infty i^2 R dt = 10 \cdot 10^{-2} \int_0^\infty e^{-5010 \cdot 2t} dt = \frac{0,1}{-10020} \cdot e^{-10020t} \Big|_0^\infty = -9,98 \cdot 10^{-6} (0 - 1) = 9,98 \text{ } \mu\text{J}$$

2.7. Svitak otpora $R = 3 \text{ } \Omega$ i induktiviteta $L = 0.6 \text{ H}$ priključuje se na izvor istosmjernog napona. U kom će trenutku struja iznositi 50 % svoje stacionarne vrijednosti? Kolika je energija izvora $E = 100\text{V}$ utrošena kroz to vrijeme na Jouleove gubitke, a kolika na uspostavljanje magnetskog polja?

$$\tau = \frac{L}{R} = \frac{0,6}{3} = 0,2 = \frac{1}{5} \text{ s}$$

$$I = \frac{E}{R} = 33,3 \text{ A}$$



$$i(t_1) = \frac{E}{2R} = \frac{E}{R} \left(1 - e^{-\frac{t_1}{\tau}} \right) \quad / \cdot 2$$

$$1 = 2 - 2 \cdot e^{-5t_1}$$

$$2e^{-5t_1} = 1$$

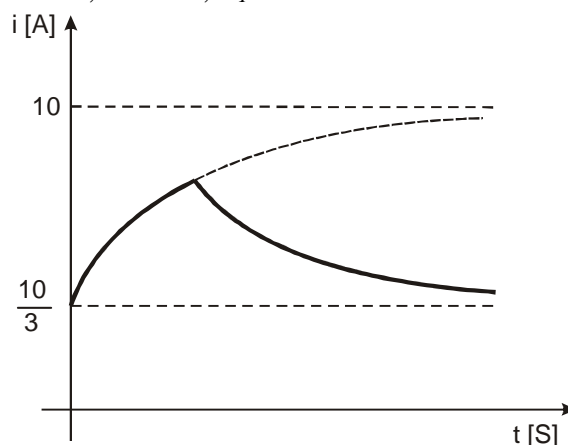
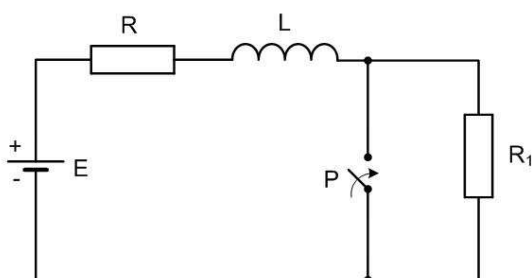
$$e^{-5t_1} = 0,5 \quad / \ln$$

$$-5t_1 = -0,6931 \quad \Rightarrow \quad t_1 = 0,1386 \text{ s}$$

$$W_L(t_1) = \frac{Li^2(t_1)}{2} = \frac{0,6}{2} \cdot \left(\frac{33,3}{2} \right)^2 = 83,3 \text{ J}$$

$$\begin{aligned} W_R(t_1) &= \int_0^{t_1} i^2 R dt = \int_0^{t_1} \frac{E^2}{R^2} \left(1 - e^{-\frac{t}{\tau}} \right)^2 R dt = \frac{E^2}{R} \int_0^{t_1} \left(1 - 2e^{-\frac{t}{\tau}} + e^{-\frac{2t}{\tau}} \right) dt = \\ &= \frac{E^2}{R} \left[\int_0^{t_1} dt - 2 \int_0^{t_1} e^{-\frac{t}{\tau}} dt + \int_0^{t_1} e^{-\frac{2t}{\tau}} dt \right] = \frac{E^2}{R} \left[t_1 - \frac{2}{-\frac{1}{\tau}} \cdot e^{-\frac{t}{\tau}} \Big|_0^{t_1} + \frac{1}{-\frac{2}{\tau}} \cdot e^{-\frac{2t}{\tau}} \Big|_0^{t_1} \right] = \\ &= \frac{E^2}{R} \left[t_1 + 2\tau \left(e^{-\frac{t_1}{\tau}} - 1 \right) - \frac{\tau}{2} \left(e^{-\frac{2t_1}{\tau}} - 1 \right) \right] = \frac{100^2}{3} \left[0,1386 + 2 \cdot 0,2 \left(e^{-\frac{0,1386}{0,2}} - 1 \right) - \frac{0,2}{2} \left(e^{-\frac{2 \cdot 0,1386}{0,2}} - 1 \right) \right] = 45,43 \text{ J} \end{aligned}$$

2.8. U strujnom krugu, prema slici, teče stacionarna struja. Ako se sklopka P u trenutku $t_0 = 0$ zatvori i nakon $t_1 = 2 \text{ s}$ ponovo otvori, odredite struju izvora od trenutka $t_0 = 0$ pa nadalje i prikažite je grafički. Zadano je: $E = 10 \text{ V}$, $R = 1 \Omega$, $L = 2 \text{ H}$, $R_1 = 2 \Omega$.



$$I = \frac{E}{R + R_1} = \frac{10}{1 + 2} = \frac{10}{3} \text{ A}$$

$$I_1 = \frac{E}{R} = \frac{10}{1} = 10 \text{ A}$$

$$\tau_a = \frac{L}{R} = \frac{2}{1} = 2 \text{ s}$$

$$i = \frac{E}{R} \left(1 - e^{-\frac{t}{\tau}} \right)$$

a) $0 \leq t \leq 2$

$$i_a = I + \frac{2}{3} \frac{E}{R} \left(1 - e^{-\frac{t}{\tau_a}} \right) = \frac{10}{3} + \frac{2 \cdot 10}{3 \cdot 1} \left(1 - e^{-0,5t} \right) = \frac{10}{3} + \frac{20}{3} - \frac{20}{3} \cdot e^{-0,5t} = 10 - \frac{20}{3} \cdot e^{-0,5t} \text{ A}$$

$$i_a(t_1) = 10 - \frac{20}{3} \cdot e^{-0,5 \cdot 2} = 7,547 \text{ A}$$

b) $2 \leq t$

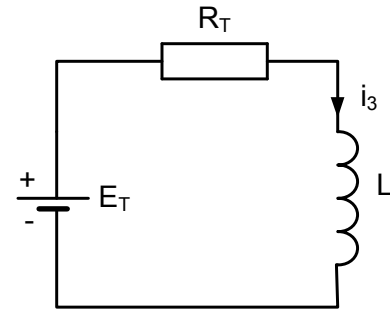
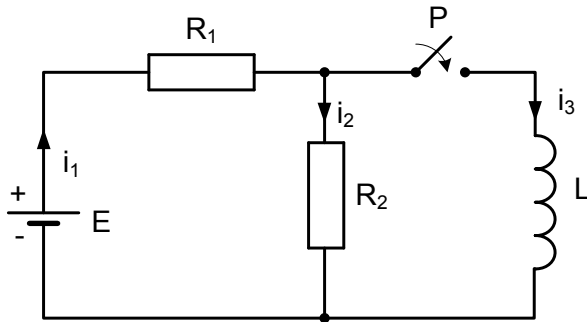
$$\tau_b = \frac{L}{R_1 + R} = \frac{2}{3} = \frac{1}{1,5} \text{ s}$$

$$\Delta I = 7,547 - \frac{10}{3} = 4,214 \text{ A}$$

$$i_b = \frac{10}{3} + 4,214 \cdot e^{-1,5(t-2)} \text{ A}$$

2.9. Treba odrediti sve struje u zadanoj mreži, prema slici, od trenutka uključivanja prekidača P pa nadalje i prikazati ih u vremenskoj ovisnosti.

Zadano je: $E = 110 \text{ V}$, $R_1 = 30 \Omega$, $R_2 = 20 \Omega$, $L = 0.12 \text{ H}$.



$$I_0 = \frac{E}{R_1 + R_2} = \frac{110}{30 + 20} = 2,2 \text{ A} \rightarrow U_2 = I_0 \cdot R_2 = 2,2 \cdot 20 = 44 \text{ V} = E_T$$

$$R_T = R_1 \parallel R_2 = 12 \Omega$$

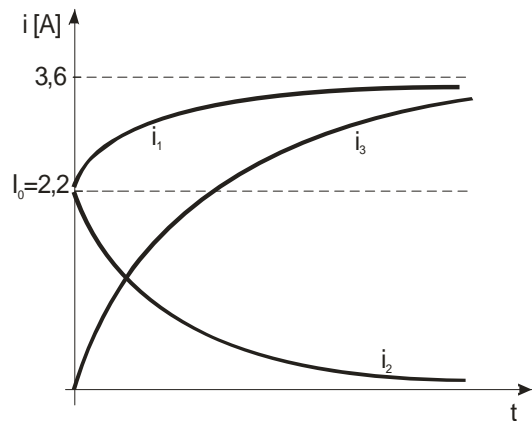
$$\tau = \frac{L}{R_T} = \frac{0,12}{12} = 0,01 \text{ s} = \frac{1}{100}$$

$$I' = \frac{E}{R_1} = \frac{110}{30} = 3,6 \text{ A}$$

$$i_3 = \frac{E_T}{R_T} \left(1 - e^{-\frac{t}{\tau}} \right) = \frac{44}{12} \left(1 - e^{-100t} \right) = 3,6 \left(1 - e^{-100t} \right) \text{ A}$$

$$i_2 = 2,2 \cdot e^{-100t} \text{ A}$$

$$i_1 = i_2 + i_3 = 2,2 \cdot e^{-100t} + 3,6 - 3,6 \cdot e^{-100t} = 3,6 - 1,46 \cdot e^{-100t} \text{ A}$$



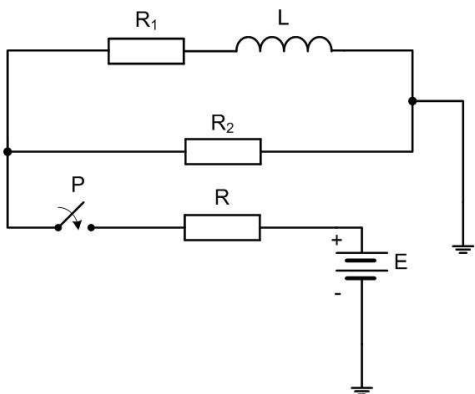
2.10. U mreži, prema slici, treba odrediti struju kroz induktivitet od trenutka uključivanja prekidača P , pa nadalje. Zadano: $E = 26 \text{ V}$, $R = 2 \Omega$, $R_1 = 1 \Omega$, $R_2 = 8 \Omega$, $L = 0,12 \text{ H}$.

$$R_T = R_1 + R \parallel R_2 = 1 + 2 \parallel 8 = 2,6 \Omega$$

$$\tau = \frac{L}{R_T} = \frac{0,12}{2,6} = 0,04615 \text{ s} = \frac{1}{21,66}$$

$$E_T = E \frac{R_2}{R + R_2} = 26 \cdot \frac{8}{2 + 8} = 20,8 \text{ V}$$

$$i_L = \frac{E_T}{R_T} \left(1 - e^{-\frac{t}{\tau}} \right) = \frac{20,8}{2,6} \left(1 - e^{-21,6t} \right) = 8 \left(1 - e^{-21,6t} \right) \text{ A}$$

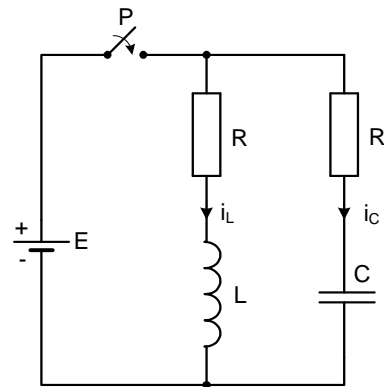


2.11. Treba odrediti vrijednost induktiviteta L , prema slici, tako da struja izvora bude konstantna od trenutka uključivanja prekidača P pa nadalje. Zadano je: $E = 100 \text{ V}$, $R = 50 \Omega$, $C = 1 \mu\text{F}$.

$$i_C = \frac{E}{R} \cdot e^{-\frac{t}{\tau_C}}$$

$$i_L = \frac{E}{R} \left(1 - e^{-\frac{t}{\tau_L}} \right) +$$

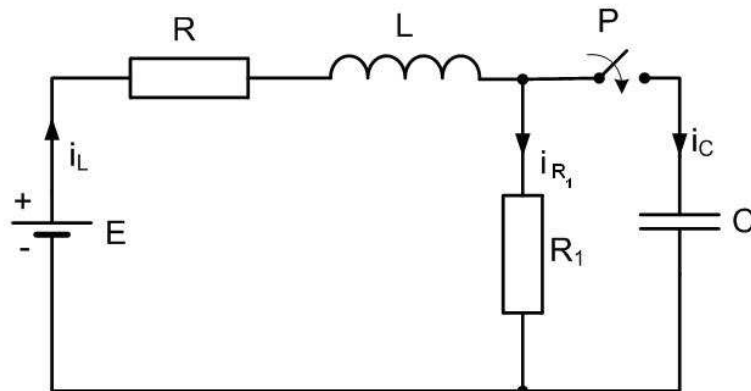
$$i = \frac{E}{R} = \text{const.}$$



$$\tau_L = \tau_C \rightarrow \frac{L}{R} = RC$$

$$L = R^2 C = 50^2 \cdot 10^{-6} = 2,5 \text{ mH}$$

2.12. Treba odrediti struje u svim granama zadane mreže, prema slici, od trenutka zatvaranja prekidača P . Koliki su napon i energija na induktivitetu L u trenutku $t = 0$, a koliki na kapacitetu C u trenutku $t = \infty$? Zadano je: $R = R_1 = 30 \Omega$, $L = 0,5 \text{ H}$, $C = 50 \mu\text{F}$, $E = 120 \text{ V}$.



$$(1) \quad i_L = i_C + i_{R_1} \rightarrow i_C = i_L - i_{R_1}$$

$$(2) \quad E = i_L \cdot R + L \frac{di_L}{dt} + i_{R_1} \cdot R_1$$

$$(3) \quad i_{R_1} \cdot R_1 = \frac{1}{C} \int i_C dt = \frac{1}{C} \int (i_L - i_{R_1}) dt \quad / \cdot C \quad / \cdot \frac{d}{dt}$$

$$i_L - i_{R_1} = R_1 C \frac{d}{dt} i_{R_1} \rightarrow i_L = R_1 C \frac{d}{dt} i_{R_1} + i_{R_1}$$

$$E = R R_1 C \frac{d}{dt} i_{R_1} + R i_{R_1} + L R_1 C \frac{d^2 i_{R_1}}{dt^2} + L \frac{di_{R_1}}{dt} + i_{R_1} R_1$$

$$L R_1 C \frac{d^2}{dt^2} i_{R_1} + (R R_1 C + L) \frac{d}{dt} i_{R_1} + i_{R_1} (R_1 - R) = E$$

$$0,5 \cdot 30 \cdot 50 \cdot 10^{-6} \cdot p^2 + (30^2 \cdot 50 \cdot 10^{-6} + 0,5) p + 60 = 0$$

$$7,5 \cdot 10^{-4} \cdot p^2 + 0,545 p + 60 = 0$$

$$p_1 = -135,27 \rightarrow \tau_1 = -\frac{1}{p_1} = 7,3924 \text{ ms}$$

$$p_2 = -591,39 \rightarrow \tau_2 = -\frac{1}{p_2} = 1,691 \text{ ms}$$

$$i_{R_1 p} = \frac{E}{R + R_1} = 2 \text{ A} = i_{L p}$$

$$i_L = A_1 e^{\frac{t}{\tau_1}} + A_2 e^{\frac{t}{\tau_2}} + 2$$

$$i_L(0_+) = A_1 + A_2 + 2 = 2 \rightarrow A_1 = -A_2$$

$$\text{za } t = 0_+ : \begin{matrix} u_{R_1} = 0 \\ u_R = 60 \text{ V} \end{matrix} \rightarrow u_L = 60 \text{ V} = L \frac{di_L}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{60}{0,5} = 120 \text{ As}$$

$$\frac{di_L}{dt} = A_1 \cdot \frac{-1}{\tau_1} \cdot e^{\frac{t}{\tau_1}} + A_2 \cdot \frac{-1}{\tau_2} \cdot e^{\frac{t}{\tau_2}}$$

$$\frac{di_L(0)}{dt} = -\frac{A_1}{\tau_1} - \frac{A_2}{\tau_2} = 120$$

$$\frac{A_2}{\tau_1} - \frac{A_2}{\tau_2} = 120 = A_2 \frac{\tau_2 - \tau_1}{\tau_1 \tau_2}$$

$$A_2 = 120 \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} = 120 \frac{7,39 \cdot 1,69 \cdot 10^{-3}}{1,691 - 7,39}$$

$$A_2 = -0,2630893 = -A_1$$

$$i_L = 0,2631 e^{\frac{t}{\tau_1}} - 0,2631 e^{\frac{t}{\tau_2}} + 2 \text{ A}$$

$$\frac{di_L}{dt} = -\frac{0,2631}{7,39 \cdot 10^{-3}} e^{\frac{t}{\tau_1}} + \frac{0,2631}{1,691 \cdot 10^{-3}} e^{\frac{t}{\tau_2}} = -35,59 e^{\frac{t}{\tau_1}} + 155,6 e^{\frac{t}{\tau_2}}$$

$$(2) \rightarrow i_{R_1} = \frac{1}{R_1} \left[E - i_L R - L \frac{di_L}{dt} \right] =$$

$$i_{R_1} = \frac{1}{30} \left[120 - \left(0,2631 e^{\frac{t}{\tau_1}} - 0,2631 e^{\frac{t}{\tau_2}} + 2 \right) \cdot 30 - 0,5 \left(-35,59 e^{\frac{t}{\tau_1}} + 155,6 e^{\frac{t}{\tau_2}} \right) \right] =$$

$$= 4 - 0,2631 e^{\frac{t}{\tau_1}} + 0,2631 e^{\frac{t}{\tau_2}} - 2 + 0,59315 e^{\frac{t}{\tau_1}} - 2,593 e^{\frac{t}{\tau_2}}$$

$$i_{R_1} = 2 + 0,33 e^{\frac{t}{\tau_1}} - 2,33 e^{\frac{t}{\tau_2}}$$

$$i_C = i_L - i_{R_1} = 2 + 0,2631 e^{\frac{t}{\tau_1}} - 0,2631 e^{\frac{t}{\tau_2}} - 2 - 0,33006 e^{\frac{t}{\tau_1}} + 2,33006 e^{\frac{t}{\tau_2}} =$$

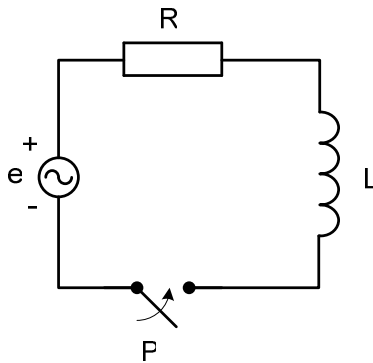
$$i_C = -0,06697 e^{\frac{t}{\tau_1}} + 2,06697 e^{\frac{t}{\tau_2}}$$

$$u_L(0) = 60 \text{ V}$$

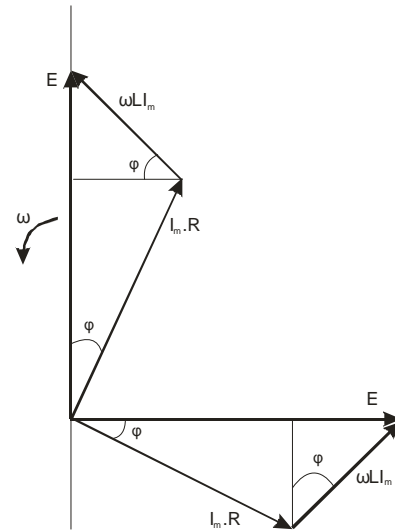
$$i_L(0) = 2 \text{ A} \rightarrow w_L(0) = \frac{i_L^2(0) \cdot L}{2} = \frac{2^2 \cdot 0,5}{2} = 1 \text{ J}$$

$$u_C(\infty) = 60 \text{ V} \rightarrow w_C(\infty) = \frac{1}{2} u_C^2(\infty) \cdot C = \frac{1}{2} \cdot 60^2 \cdot 50 \cdot 10^{-6} = 0,09 \text{ J}$$

2.13. Serijski spoj otpora $R = 10 \, \Omega$ i induktiviteta $L = 0.1 \, \text{H}$, priključen je na sinusnu EMS $e = 10 \sin(500t) \, \text{V}$. Treba odrediti struju uključenja.



$$\begin{aligned} u_L + u_R &= e \\ L \frac{di}{dt} + Ri &= e \\ Lp + R &= 0 \\ p &= -\frac{R}{L} = -\frac{10}{0,1} = -100 \\ \tau &= -\frac{1}{p} = 0,01 \, \text{s} \end{aligned}$$



$$i = i_s + i_p = Ae^{-\frac{t}{\tau}} + I_m \sin(\omega t - \varphi)$$

$$L \frac{di_s}{dt} + Ri_s + \omega LI_m \cos(\omega t - \varphi) + RI_m \sin(\omega t - \varphi) = E \sin \omega t$$

$$(1) \quad L \frac{di_s}{dt} + Ri_s = 0$$

$$(2) \quad \omega LI_m \cos(\omega t - \varphi) + RI_m \sin(\omega t - \varphi) = E \sin \omega t, \quad \cos(\omega t - \varphi) = \sin(\omega t - \varphi + 90^\circ)$$

$$\omega t_1 = 0 \rightarrow (2) \rightarrow \omega LI_m \cos \varphi - RI_m \sin \varphi = 0 \rightarrow$$

$$\frac{\sin \varphi}{\cos \varphi} = \tan \varphi = \frac{\omega L}{R} = \frac{500 \cdot 0,1}{10} = 5 \rightarrow \varphi = 78,65^\circ$$

$$\omega t_2 = \frac{\pi}{2} \rightarrow (2) \rightarrow I_m [\omega L \sin \varphi + R \cos \varphi] = E_m$$

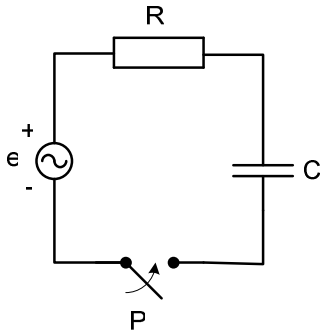
$$(2) \quad E = \sqrt{(\omega LI_m)^2 + (RI_m)^2 + 2\omega LI_m \cos(\omega t - \varphi - \omega t + \varphi - 90^\circ)}$$

$$= I_m \sqrt{(\omega L)^2 + R^2} \rightarrow I_m = \frac{E}{\sqrt{R^2 + \omega^2 L^2}} = \frac{10}{\sqrt{10^2 + (500 \cdot 0,1)^2}} = \frac{10}{50,99} = 0,1961 \, \text{A}$$

$$i(0) = 0,1961 \sin(-78,69^\circ) + Ae^{-100 \cdot 0} = 0 \rightarrow A = 0,19231$$

$$i(t) = 0,1961 \sin(500t - 78,69^\circ) + 0,1923e^{-100t} \, \text{A}$$

2.14. Serijski spoj otpora $R = 100 \, \Omega$ i kapaciteta $C = 20 \, \mu\text{F}$, priključen je na EMS $e = 100 \sin(500t + 30^\circ) \, \text{V}$. Treba odrediti struju uključenja.



$$X_C = \frac{1}{\omega C} = 100 \, \Omega$$

$$\frac{1}{C} \int i dt + u_C(0) + R i = e \quad \therefore \frac{d}{dt} \rightarrow R \frac{di}{dt} + \frac{1}{C} i = \frac{d}{dt} e$$

$$R p + \frac{1}{C} = 0$$

$$p = -\frac{1}{RC}$$

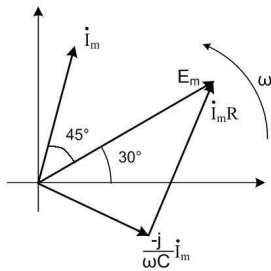
$$\tau = -\frac{1}{p} = RC = 100 \cdot 20 \cdot 10^{-6} = 0,002 = \frac{1}{500} \, s$$

$$i = i_s + I_m \sin(\omega t - \varphi)$$

$$i_p \rightarrow \frac{1}{C} \int i dt + R i = e$$

$$-\frac{I_m}{\omega C} \cos(\omega t - \varphi) + R I_m \sin(\omega t - \varphi) = E_m \sin(\omega t + 30^\circ)$$

$$-\frac{I_m}{\omega C} \sin(\omega t - \varphi + 90^\circ) + R I_m \sin(\omega t - \varphi) = E_m \sin(\omega t + 30^\circ)$$



za

$$\omega t + 30^\circ = 0 \rightarrow \omega t = -30^\circ \rightarrow -\frac{I_m}{\omega C} \cos(\varphi + 30^\circ) - R I_m \sin(\varphi + 30^\circ) = 0$$

$$\frac{\sin(\varphi + 30^\circ)}{\cos(\varphi + 30^\circ)} = \frac{-I_m}{\omega C R I_m} = -\frac{1}{\omega C R} = \tan(\varphi + 30^\circ) = -1$$

$$\varphi + 30^\circ = -45^\circ \rightarrow \varphi = -75^\circ$$

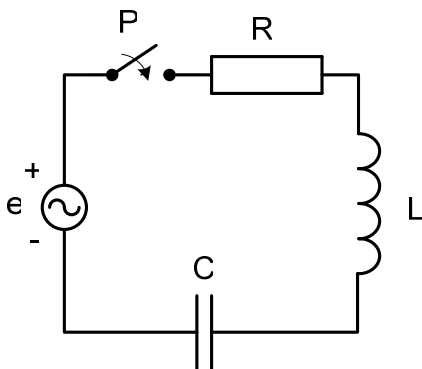
$$E_m = \sqrt{\left(\frac{I_m}{\omega C}\right)^2 + (R I_m)^2 + 2 \frac{I_m}{\omega C} R I_m \cdot \cos(\omega t - \varphi - \omega t + \varphi - 90^\circ)} = I_m \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$I_m = \frac{E_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{100}{\sqrt{100^2 + \left(\frac{1}{500 \cdot 20 \cdot 10^{-6}}\right)^2}} = \frac{100}{100\sqrt{2}} = 0,707 \, A$$

$$i(0) = 0,707 \sin 75^\circ + A \cdot e^{-500 \cdot 0} = \frac{E_m \sin 30^\circ}{R} = \frac{100 \cdot 0,5}{100} = 0,5 \rightarrow A = 0,5 - 0,683 = -0,183$$

$$i(t) = 0,707 \sin(\omega t + 75^\circ) - 0,183 \cdot e^{-500t} \, A$$

2.15. Serijski spoj RLC ($R = 100 \Omega$, $L = 0,1 \text{ H}$ i $C = 100 \mu\text{F}$) priključen je na EMS
 $e = 100 \cos(1000t) \text{ V}$. Treba odrediti struju uključenja.



$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = e \quad / \frac{d}{dt}$$

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = \frac{de}{dt}$$

$$i = i_s + I_m \cos(\omega t - \varphi)$$

$$\cos \alpha = \sin(90^\circ + \alpha)$$

$$\cos(90^\circ + \alpha) = -\sin \alpha$$

slobodni odziv:

$$Lp^2 + Rp + \frac{1}{C} = 0$$

$$0,1p^2 + 100p + \frac{1}{100 \cdot 10^{-6}} = 0$$

$$p_1 = -112,702 \rightarrow \tau_1 = -\frac{1}{p_1} = 8,873 \text{ ms} = \frac{1}{112,7}$$

$$p_2 = -887,3 \rightarrow \tau_2 = -\frac{1}{p_2} = 1,127 \text{ ms} = \frac{1}{887,3}$$

prisilni odziv:

$$-\omega LI_m \sin(\omega t - \varphi) + \frac{I_m}{\omega C} \sin(\omega t - \varphi) + RI_m \cos(\omega t - \varphi) = E \cos \omega t$$

$$\text{za } \omega t = \frac{\pi}{2}: -\omega LI_m \cos \varphi + \frac{I_m}{\omega C} \cos \varphi + RI_m \sin \varphi = 0$$

$$RI_m \sin \varphi = \cos \varphi \left[\omega L - \frac{1}{\omega C} \right] I_m \quad / : I_m$$

$$\frac{\sin \varphi}{\cos \varphi} = \frac{\omega L - \frac{1}{\omega C}}{R} = \text{tg } \varphi = \frac{10^3 \cdot 0,1 - \frac{1}{10^3 \cdot 10^2 \cdot 10^{-6}}}{100} = 0,9 \rightarrow \varphi = 41,99^\circ$$

$$\text{za } \omega t = 0: \omega LI_m \sin \varphi - \frac{I_m}{\omega C} \sin \varphi + RI_m \cos \varphi = E$$

$$I_m \left[\omega L - \frac{1}{\omega C} \right] \sin \varphi + RI_m \sin[\varphi + 90^\circ] = E$$

$$E = \sqrt{I_m^2 \left[\omega L - \frac{1}{\omega C} \right]^2 + R^2 I_m^2} + 2I_m \left(\omega L - \frac{1}{\omega C} \right) RI_m \cos(\varphi + 90^\circ - \varphi) = I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$I_m = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{100}{\sqrt{100^2 + \left(10^3 \cdot 0,1 - \frac{1}{10^3 \cdot 10^2 \cdot 10^{-6}}\right)^2}} = \frac{100}{134,54} = 0,7433 \text{ A}$$

$$i(0) = 0,7433 \cos(-41,99^\circ) + A_1 e^{-112,7 \cdot 0} + A_2 e^{-887,3 \cdot 0} = 0 \rightarrow A_1 = -A_2 - 0,5525$$

$$i(t) = I_m \cos(\omega t - \varphi) + A_1 e^{-p_{t_1} t} + A_2 e^{-p_{t_2} t} = 0,7433 \cos(\omega t - 41,99^\circ) + A_1 e^{-112,7 t} + A_2 e^{-887,3 t}$$

$$\frac{di}{dt} = -0,7433 \cdot 1000 \sin(\omega t - 41,99^\circ) + A_1 (-112,7) e^{-112,7 t} + A_2 (-887,3) e^{-887,3 t}$$

$$\frac{di}{dt}(0) = -743,3 \sin(-41,99^\circ) - 112,7 A_1 - 887,3 A_2 = \frac{E_m}{L} = \frac{100}{0,1} = 10^3$$

$$-112,7(-A_2 - 0,5525) - 887,3 A_2 = 10^3 - 497,3$$

$$A_2 (112,7 - 887,3) = 10^3 - 497,3 - 112,7 \cdot 0,5525$$

$$A_2 = -0,5687 \rightarrow A_1 = 0,0162$$

$$i(t) = 0,7433 \cos(1000t - 41,99^\circ) + 0,0162 \cdot e^{-112,7 t} - 0,5687 \cdot e^{-887,3 t}, A$$

3. Prikazivanje sinusnih veličina pomoću fazora

3.1. Treba napisati sljedeće kompleksne brojeve u eksponencijalnom obliku: a) $3 + j4$; b) $3 - j4$; c) $-3 - j4$; d) $-3 + j4$; e) $0,048 - j0,153$; f) $0,0171 + j0,047$.

$$a) \quad 3 + j4 = \sqrt{3^2 + 4^2} \left| \arctg \frac{4}{3} \right| = 5 \left| 53,13^\circ \right|$$

$$b) \quad 3 - j4 = \sqrt{3^2 + 4^2} \left| \arctg \frac{-4}{3} \right| = 5 \left| -53,13^\circ \right|$$

$$c) \quad -3 - j4 = 5 \left| \arctg \frac{-4}{-3} \right| = 5 \left| -126,87^\circ \right|$$

$$d) \quad -3 + j4 = 5 \left| \arctg \frac{4}{-3} \right| = 5 \left| 126,87^\circ \right|$$

$$e) \quad 0,048 - j0,153 = 0,16035 \left| -72,582^\circ \right|$$

$$f) \quad 0,0171 + j0,047 = 0,05001 \left| 70,007^\circ \right|$$

3.2. Treba napisati sljedeće kompleksne brojeve u Kartezijevu obliku: a) $12,3 \left| 30^\circ \right|$; b) $53 \left| 160^\circ \right|$; c) $25 \left| -45^\circ \right|$; d) $86 \left| -115^\circ \right|$; e) $0,05 \left| -20^\circ \right|$; f) $0,013 \left| 260^\circ \right|$; g) $0,156 \left| -190^\circ \right|$; h) $200 \left| 181^\circ \right|$.

$$a) \quad 12,3 \left| 30^\circ \right| = 12,3(\cos 30^\circ + j \sin 30^\circ) = 10,652 + j6,15 = 6,15(\sqrt{3} + j)$$

$$b) \quad 53 \left| 160^\circ \right| = -49,803 + j18,127$$

$$c) \quad 25 \left| -45^\circ \right| = 17,678 - j17,678$$

$$d) \quad 86 \left| -115^\circ \right| = -36,345 - j77,942$$

$$e) \quad 0,05 \left| -20^\circ \right| = 0,04698 - j0,01710$$

$$f) \quad 0,013 \left| 260^\circ \right| = -0,002257 - j0,01280$$

$$g) \quad 0,156 \left| -190^\circ \right| = -0,15363 + j0,027089$$

$$h) \quad 200 \left| 181^\circ \right| = -199,9695 - j3,49048$$

3.3. Treba odrediti sumu i razliku sljedećih kompleksnih brojeva: a) $10 \left| 53,1^\circ \right| + 4 + j2$;

$$b) \quad 10 \left| 90^\circ \right| + (8 - j2); \quad c) \quad (-4 - j6) + 2 + j4; \quad d) \quad 2,83 \left| 45^\circ \right| - (2 - j8); \quad e) \quad (-5 + j5) - 7,07 \left| 135^\circ \right|;$$

$$f) \quad -5 \left| 53,1^\circ \right| - (1 - j6).$$

$$a) \quad 10 \left| 53,1^\circ \right| + 4 + j2 = 6 + j8 + 4 + j2 = 10 + j10$$

$$b) \quad 10 \left| 90^\circ \right| + (8 - j2) = j10 + 8 - j2 = 8 + j8$$

$$c) \quad (-4 - j6) + 2 + j4 = -2 - j2$$

$$d) \quad 2,83 \left| 45^\circ \right| - (2 - j8) = 2 + j2 - 2 + j8 = j10$$

$$e) (-5 + j5) - 7,07 \angle 135^\circ = -5 + j5 - (-5 + j5) = 0$$

$$f) -5 \angle 53,1^\circ - (1 - j6) = -(3 + j4) - 1 + j6 = -4 + j2$$

3.4. Treba odrediti produkt sljedećih izraza: a) $(3 - j2)(1 - j4)$; b) $(-1 - j)(1 + j)$;

$$c) j2(4 - j3); d) (-j6)(j6); e) (-4 + j6) \cdot 5 \angle 53,1^\circ; f) 10 \angle 90^\circ \cdot (-j2);$$

$$g) 10 \angle 53,1^\circ \cdot 10 \angle -23^\circ.$$

$$a) (3 - j2)(1 - j4) = 3,605 \angle -33,69^\circ \cdot 4,1231 \angle -75,96^\circ = 14,866 \angle -109,6^\circ = -5 - j14$$

$$b) (-1 - j)(1 + j) = \sqrt{2} \angle -135^\circ \cdot \sqrt{2} \angle 45^\circ = 2 \angle -90^\circ = -j2$$

$$c) j2(4 - j3) = 2 \angle 90^\circ \cdot 5 \angle -36,87^\circ = 10 \angle 53,13^\circ = 6 + j8$$

$$d) (-j6)(j6) = -36 j^2 = 36$$

$$e) (-4 + j6) \cdot 5 \angle 53,1^\circ = 7,211 \angle 123,69^\circ \cdot 5 \angle 53,1^\circ = 36,055 \angle 176,79^\circ = -36 + j2$$

$$f) 10 \angle 90^\circ \cdot (-j2) = j10(-j2) = 20$$

$$g) 10 \angle 53,1^\circ \cdot 10 \angle -23^\circ = 100 \angle 30,1^\circ = 86,515 + j50,151$$

3.5. Treba odrediti kvocijent sljedećih izraza:

$$a) \frac{4 - j8}{2 + j2}; b) \frac{-5 + j2}{4 - j3}; c) \frac{5 - j10}{3 + j4}; d) \frac{10}{3 + j4}; e) \frac{j5}{2 - j2} \cdot 3 \angle -30^\circ; f) \frac{6,88 \angle 12^\circ}{2 + j6};$$

$$g) \frac{25 \angle 30^\circ}{(3 + j4) \cdot 5 \angle -30^\circ}; h) \frac{1}{6 + j8} \cdot (3 - j2) : 10e^{-j\frac{2\pi}{3}}.$$

$$a) \frac{4 - j8}{2 + j2} = \frac{8,944 \angle -63,43^\circ}{2,828 \angle 45^\circ} = 3,162 \angle -108,43^\circ = -1 - j3$$

$$b) \frac{-5 + j2}{4 - j3} = \frac{5,385 \angle 158,199^\circ}{5 \angle -36,87^\circ} = 1,077 \angle 195^\circ = -1,04 - j0,28$$

$$c) \frac{5 - j10}{3 + j4} = \frac{11,18 \angle -63,43^\circ}{5 \angle 53,13^\circ} = 2,236 \angle -116,57^\circ = -1 - j2$$

$$d) \frac{10}{3 + j4} = \frac{10}{5 \angle 53,13^\circ} = 2 \angle -53,13^\circ = 1,2 - j1,6$$

$$e) \frac{j5 \cdot 3 \angle -30^\circ}{2 - j2} = \frac{15 \angle 60^\circ}{2\sqrt{2} \angle -45^\circ} = 5,303 \angle 105^\circ = -1,373 + j5,122$$

$$f) \frac{6,88 \angle 12^\circ}{2 + j6} = \frac{6,88 \angle 12^\circ}{6,324 \angle 71,56^\circ} = 1,088 \angle -59,56^\circ = 0,551 - j0,938$$

$$g) \frac{25 \angle 30^\circ}{(3 + j4) \cdot 5 \angle -30^\circ} = \frac{5 \angle 60^\circ}{5 \angle 53,13^\circ} = 1 \angle 6,87^\circ = 0,9928 + j0,1196$$

$$h) \frac{3 - j2}{(6 + j8) \cdot 10 \angle -120^\circ} = \frac{3,605 \angle -33,69^\circ}{10 \angle 53,13^\circ \cdot 10 \angle -120^\circ} = 0,03605 \angle 33,18^\circ = 0,03018 + j0,01973$$

3.6. Treba odrediti sljedeće izraze: $\dot{A} \cdot \dot{A}^*$, $\dot{A} + \dot{A}^*$, $\dot{A} - \dot{A}^*$, \dot{A} / \dot{A}^* , ako je

$$a) \dot{A} = 3 - j4; b) \dot{A} = 10 \angle -30^\circ; c) \dot{A} = 2,5e^{-j\frac{\pi}{3}}.$$

$$\begin{aligned} a) \quad & (3 - j4)(3 + j4) = 3^2 + 4^2 = 25 \\ & 3 - j4 + 3 + j4 = 6 \\ & 3 - j4 - 3 - j4 = -j8 \\ & \frac{3 - j4}{3 + j4} = \frac{5 \angle -53,13^\circ}{5 \angle 53,13^\circ} = 1 \angle -106,26^\circ = -0,28 - j0,96 \end{aligned}$$

$$\begin{aligned} b) \quad & 10 \angle -30^\circ \cdot 10 \angle 30^\circ = 100 \\ & 10 \angle -30^\circ + 10 \angle 30^\circ = 17,32 \\ & 10 \angle -30^\circ - 10 \angle 30^\circ = 8,66 - j5 - 8,66 - j5 = -j10 \\ & \frac{10 \angle -30^\circ}{10 \angle 30^\circ} = 1 \angle -60^\circ \end{aligned}$$

$$\begin{aligned} c) \quad & 2,5 \angle -60^\circ \cdot 2,5 \angle 60^\circ = 6,25 \\ & 2,5 \angle -60^\circ + 2,5 \angle 60^\circ = 1,25 - j2,165 + 1,25 + j2,165 = 2,5 \\ & 2,5 \angle -60^\circ - 2,5 \angle 60^\circ = 1,25 - j2,165 - 1,25 - j2,165 = -j4,33 \\ & \frac{2,5 \angle -60^\circ}{2,5 \angle 60^\circ} = 1 \angle -120^\circ \end{aligned}$$

3.7. Treba odrediti sljedeći izraz: $\frac{\dot{A} \cdot \dot{B}}{\dot{A} + \dot{B}}$ ako je: a) $\dot{A} = 10 + j5$, $\dot{B} = 20 \angle 30^\circ$;

$$b) \dot{A} = 6 - j2, \dot{B} = 1 + j8; c) \dot{A} = 10 \angle 45^\circ, \dot{B} = 5 \angle 70^\circ; d) \dot{A} = -j5, \dot{B} = j10$$

$$e) \dot{A} = 10, \dot{B} = -j10.$$

$$\begin{aligned}
a) \quad & \frac{11,18 \angle 26,565^\circ \cdot 20 \angle 30^\circ}{10 + j5 + 17,32 + j10} = \frac{223,6 \angle 56,565^\circ}{27,32 + j15} = \frac{223,6 \angle 56,565^\circ}{31,167 \angle 28,768^\circ} = 7,174 \angle 27,797^\circ \\
b) \quad & \frac{6,324 \angle -18,43^\circ \cdot 8,062 \angle 82,87^\circ}{6 - j2 + 1 + j8} = \frac{50,99 \angle 64,44^\circ}{9,2195 \angle 40,60^\circ} = 5,531 \angle 23,84^\circ \\
c) \quad & \frac{10 \angle 45^\circ \cdot 5 \angle 70^\circ}{7,07 + j7,07 + 1,71 + j4,7} = \frac{50 \angle 115^\circ}{8,781 + j11,77} = \frac{50 \angle 115^\circ}{14,68 \angle 53,27^\circ} = 3,4 \angle 61,73^\circ = 1,613 + j3 \\
d) \quad & \frac{-j5 \cdot j10}{-j5 + j10} = \frac{50}{5 \angle 90^\circ} = 10 \angle -90^\circ = -j10 \\
e) \quad & \frac{10 \cdot (-j10)}{10 - j10} = \frac{-j100}{10\sqrt{2} \angle -45^\circ} = \frac{100 \angle -90^\circ}{14,142 \angle -45^\circ} = 7,07 \angle -45^\circ
\end{aligned}$$

3.8. Treba napisati fazore sljedećih funkcija:

$$\begin{aligned}
a) \quad & i = 100 \sin\left(\omega t - \frac{\pi}{3}\right); \quad b) \quad u = 200 \sin(\omega t + 180^\circ); \quad c) \quad e = 10\sqrt{2} \sin(\omega t - 90^\circ); \\
d) \quad & i = \sqrt{2} \cdot 5 \sin(\omega t + 120^\circ); \quad e) \quad i = \sqrt{2} \cdot 10 \sin(\omega t + 240^\circ); \quad f) \quad i = \sqrt{2} \sin \omega t.
\end{aligned}$$

$$a) \quad \tilde{i} = \sqrt{2} \dot{I} e^{-j\omega t}; \quad \dot{I} = \frac{100}{\sqrt{2}} e^{-j60^\circ}$$

$$b) \quad \tilde{u} = \sqrt{2} \dot{U} e^{j\omega t}; \quad \dot{U} = \frac{200}{\sqrt{2}} e^{j180^\circ}$$

$$c) \quad \tilde{e} = \sqrt{2} \dot{E} e^{j\omega t}; \quad \dot{E} = 10 e^{-j90^\circ}$$

$$d) \quad \tilde{i} = \sqrt{2} \dot{I} e^{j\omega t}; \quad \dot{I} = 5 e^{j120^\circ}$$

$$e) \quad \tilde{i} = \sqrt{2} \dot{I} e^{j\omega t}; \quad \dot{I} = 10 e^{j240^\circ}$$

$$f) \quad \tilde{i} = \sqrt{2} \dot{I} e^{j\omega t}; \quad \dot{I} = 1 \angle 0^\circ$$

3.9. Treba odrediti struju $i = i_1 + i_2$ ako je: $i_1 = \sqrt{2} \cdot 5 \sin\left(\omega t + \frac{2\pi}{3}\right)$, $i_2 = \sqrt{2} \cdot 5 \sin\left(\omega t + \frac{4\pi}{3}\right)$.

$$\dot{I}_1 = 5 \angle 120^\circ = -2,5 + j4,33$$

$$\dot{I}_2 = 5 \angle 240^\circ = -2,5 - j4,33$$

$$\dot{I}_1 + \dot{I}_2 = -5 \rightarrow i = 5\sqrt{2} \sin(\omega t - \pi)$$

3.10. Treba odrediti: $e = e_1 + e_2 + e_3$, ako je $e_1 = 220\sqrt{2} \sin \omega t$, $e_2 = 220\sqrt{2} \sin\left(\omega t - \frac{2\pi}{3}\right)$,

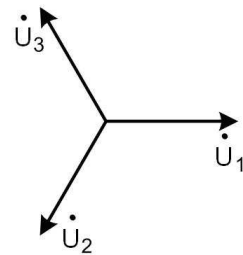
$$e_3 = 220\sqrt{2} \sin\left(\omega t + \frac{2\pi}{3}\right).$$

$$\dot{E}_1 = 220$$

$$\dot{E}_2 = 220 \angle -120^\circ = -110 - j190,52$$

$$\dot{E}_3 = 220 \angle 120^\circ = -110 + j190,52$$

$$\dot{E}_1 + \dot{E}_2 + \dot{E}_3 = 0, \quad e = 0$$



3.11. Treba odrediti $u = u_1 + u_2$ ako je $u_1 = 10 \sin(\omega t + 30^\circ)$, $u_2 = 10 \cos(\omega t - 45^\circ)$.

$$\dot{U}_1 = \frac{10}{\sqrt{2}} \angle 30^\circ = 6,1237 + j3,5355$$

$$u_2 = 10 \sin(\omega t - 45^\circ + 90^\circ) = 10 \sin(\omega t + 45^\circ)$$

$$\dot{U}_2 = \frac{10}{\sqrt{2}} \angle 45^\circ = 5 + j5$$

$$\dot{U}_1 + \dot{U}_2 = 11,1237 + j8,5355 = 14,02 \angle 37,5^\circ$$

$$u = 14,02\sqrt{2} \sin(\omega t + 37,5^\circ) = 19,83 \sin(\omega t + 37,5^\circ) \quad V$$

3.12. Treba odrediti $i = i_1 - i_2$ ako je $i_1 = \sqrt{2} \sin(\omega t + 45^\circ)$, a $i_2 = \sqrt{2} \cos(\omega t - 45^\circ)$.

$$\dot{I}_1 = 1 \angle 45^\circ$$

$$i_2 = \sqrt{2} \sin(\omega t - 45^\circ + 90^\circ) = \sqrt{2} \sin(\omega t + 45^\circ)$$

$$\dot{I}_2 = 1 \angle 45^\circ$$

$$i = i_1 - i_2 = 0$$

3.13. Zadana je frekvencija i fazor struje. Treba napisati struju u vremenskom obliku:

a) $\omega = 100 \text{ rad/s}$, $\dot{I} = 10 \angle 30^\circ \text{ A}$,

b) $\omega = 300 \text{ rad/s}$, $\dot{I} = 2 \angle 45^\circ \text{ A}$,

c) $\omega = 400 \text{ rad/s}$, $\dot{I} = 3 \angle 45^\circ \text{ A}$,

d) $\omega = 500 \text{ rad/s}$, $\dot{I} = 2 - j3 \text{ A}$,

e) $\omega = 600 \text{ rad/s}$, $\dot{I} = -2 - j3 \text{ A}$.

Rješenja:

a) $i = 10\sqrt{2} \sin(100t + 30^\circ) \text{ A}$

b) $i = 2\sqrt{2} \sin(300t + 45^\circ) \text{ A}$

c) $i = 3\sqrt{2} \sin(400t + 45^\circ) \text{ A}$

d) $i = 3,605\sqrt{2} \sin(500t - 56,31^\circ) \text{ A}$

e) $i = 5,1 \sin(600t + 236,31^\circ) \text{ A}$

3.14. Zadano je $i = 10 \sin(200t - 30^\circ) \text{ A}$, treba odrediti pomoću rotirajućeg fazora

derivaciju $\frac{di}{dt}$ i integral $\int i dt$.

$$\dot{I}_m = 10 \angle -30^\circ \text{ A} \quad \Rightarrow \quad j\omega \dot{I}_m = 200 \angle 90^\circ \cdot 10 \angle -30^\circ = 2000 \angle 60^\circ$$

$$\frac{di}{dt} = 2000 \sin(200t + 60^\circ) \text{ A/s}$$

$$\frac{\dot{I}_m}{j\omega} = \frac{10 \angle -30^\circ}{200 \angle 90^\circ} = 0,05 \angle -120^\circ \Rightarrow \int i dt = 0,05 \sin(200t + 240^\circ) \text{ As}$$

3.15. Zadan je napon $u = \sqrt{2} \cdot 220 \sin(314t + \frac{\pi}{6}) \text{ V}$, treba odrediti struju u realnom obliku i

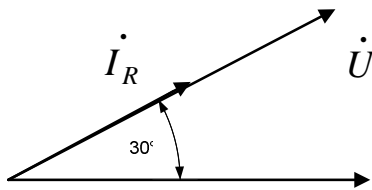
fazorskom obliku: a) otpora $R = 10 \Omega$; b) induktiviteta $L = 3 \text{ mH}$ i c) kapaciteta $C = 3 \text{ mF}$. Fazore napona i struja prikazati grafički.

$$\dot{U} = 220 \angle 30^\circ \text{ V};$$

$$X_L = \omega L = 314 \cdot 3 \cdot 10^{-3} = 0,9425 \Omega$$

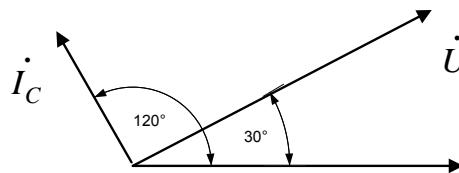
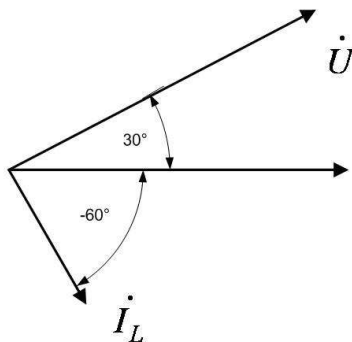
$$X_C = \frac{1}{\omega C} = \frac{1}{314 \cdot 3 \cdot 10^{-3}} = 1,061 \Omega$$

$$\dot{I}_R = \frac{\dot{U}}{R} = \frac{220 \angle 30^\circ}{10} = 22 \angle 30^\circ \text{ A}$$



$$\dot{I}_L = \frac{220 \angle 30^\circ}{0,9425 \angle 90^\circ} = 233,4 \angle -60^\circ = \frac{330,1}{\sqrt{2}} \angle -60^\circ \text{ A}$$

$$\dot{I}_C = \frac{220 \angle 30^\circ}{1,061 \angle -90^\circ} = 207,3 \angle 120^\circ = \frac{293,2}{\sqrt{2}} \angle 120^\circ \text{ A}$$



3.16. Zadana je struja $i = \sqrt{2} \cdot 10 \sin(1000t - \frac{\pi}{6})$ A, treba odrediti pomoću fazora napon na: a) otporu $R = 10 \Omega$; b) induktivitetu $L = 10 \text{ mH}$; c) kapacitetu $C = 0,1 \text{ mF}$ i nacrtati fazorski dijagram struje i napona.

$$\dot{I} = 10 \angle -30^\circ \text{ A};$$

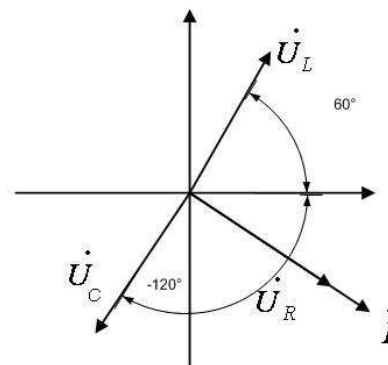
$$X_L = \omega L = 10^3 \cdot 10^{-2} = 10 \Omega;$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^3 \cdot 10^{-4}} = 10 \Omega;$$

$$\dot{U}_R = \dot{I} \cdot R = 10 \angle -30^\circ \cdot 10 = 100 \angle -30^\circ \text{ V}$$

$$\dot{U}_L = jX_L \cdot \dot{I} = 10 \angle 90^\circ \cdot 10 \angle -30^\circ = 100 \angle 60^\circ \text{ V}$$

$$\dot{U}_C = -jX_C \cdot \dot{I} = 10 \angle -90^\circ \cdot 10 \angle -30^\circ = 100 \angle -120^\circ \text{ V}$$



3.17. Treba odrediti trenutnu snagu, radnu i jalovu snagu te trenutnu energiju u zadatku 3.15 i nacrtati sve trenutne veličine.

$$u = U_m \cdot \sin(\omega t + \alpha_u) ; \quad i = I_m \cdot \sin(\omega t + \alpha_i)$$

$$p(t) = u \cdot i = U_m I_m \cdot \sin(\omega t + \alpha_u) \cdot \sin(\omega t + \alpha_i) =$$

$$= \frac{U_m \cdot I_m}{2} \cdot [\cos(\omega t + \alpha_u - \omega t - \alpha_i) - \cos(\omega t + \alpha_u + \omega t + \alpha_i)]$$

$$p(t) = U \cdot I \cdot [\cos \varphi - \cos(2\omega t + \varphi + 2\alpha_i)] \quad (\alpha_u + \alpha_i + \alpha_i - \alpha_i = \varphi + 2\alpha_i)$$

$$a) \quad \varphi_R = 0 \Rightarrow$$

$$p_R(t) = u \cdot i_R = 220 \cdot 22 \cdot [\cos 0^\circ - \cos(2 \cdot 314t + 0^\circ + 60^\circ)] = 4840[1 - \cos(628t + 60^\circ)]$$

$$P = 4840 \text{ W} ; \quad Q = 0 = U \cdot I \cdot \sin \varphi_R$$

$$w_R(t) = \int p_R(t) dt = \int [4840 - 4840 \cos(628t + 60^\circ)] \cdot dt = 4840t - \frac{4840}{628} \cdot \sin(628t + 60^\circ) =$$

$$= 4840t - 7,707 \cdot \sin(628t + 60^\circ) \text{ J}$$

$$b) \quad \varphi_L = 90^\circ \Rightarrow \quad p_L(t) = u \cdot i_L = 220 \cdot 233,4 \cdot [\cos 90^\circ - \cos(2\omega t + 90^\circ - 120^\circ)] =$$

$$= 51354 \cdot [0 - \cos(2\omega t - 30^\circ)] = -51354 \cdot \cos(2\omega t - 30^\circ) =$$

$$= 51354 \cdot \cos(2\omega t + 150^\circ)$$

$$P = 0 \quad Q = 51354 \text{ VAR}$$

$$w_L(0) = \frac{1}{2} L \cdot i_L^2(0) = \frac{1}{2} \cdot 3 \cdot 10^{-3} \cdot [330,1 \cdot \sin(-60^\circ)]^2 = 122,7 \text{ J}$$

$$w_L(t) = \int p_L(t) dt + K_1 = \int 51354 \cdot \cos(2\omega t + 150^\circ) dt + K_1 = \frac{51354}{628} \cdot \sin(2\omega t + 150^\circ) + K_1 =$$

$$= 81,77 \cdot \sin(2\omega t + 150^\circ) + K_1$$

$$w_L(0) = 122,7 \Rightarrow K_1 = 122,7 - 81,77 \cdot \sin 150^\circ = 81,7 \text{ J}$$

$$w_L(t) = 81,77 \cdot \sin(2\omega t + 150^\circ) + 81,7 \text{ J}$$

$$c) \quad \varphi_C = -90^\circ \Rightarrow \quad p_C(t) = u \cdot i_C = 220 \cdot 207,3 \cdot [\cos(-90^\circ) - \cos(2\omega t - 90^\circ + 240^\circ)]$$

$$p_C(t) = 45616 \cdot [0 - \cos(2\omega t + 150^\circ)] = -45616 \cdot \cos(2\omega t + 150^\circ) =$$

$$= 45616 \cdot \cos(2\omega t - 30^\circ)$$

$$w_C(0) = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot 3 \cdot 10^{-3} \cdot (220\sqrt{2} \cdot \sin 30^\circ)^2 = 36,3 \text{ J}$$

$$w_C(t) = \int p_C(t) dt + K_2 = \int 45616 \cdot \cos(2\omega t - 30^\circ) dt + K_2 = \frac{45616}{628} \cdot \sin(2\omega t - 30^\circ) + K_2 =$$

$$= 72,64 \cdot \sin(2\omega t - 30^\circ) + K_2 \text{ J} \quad \rightarrow \quad K_2 = 36,3 - 72,64 \cdot \sin(-30^\circ) = 72,62 \text{ J}$$

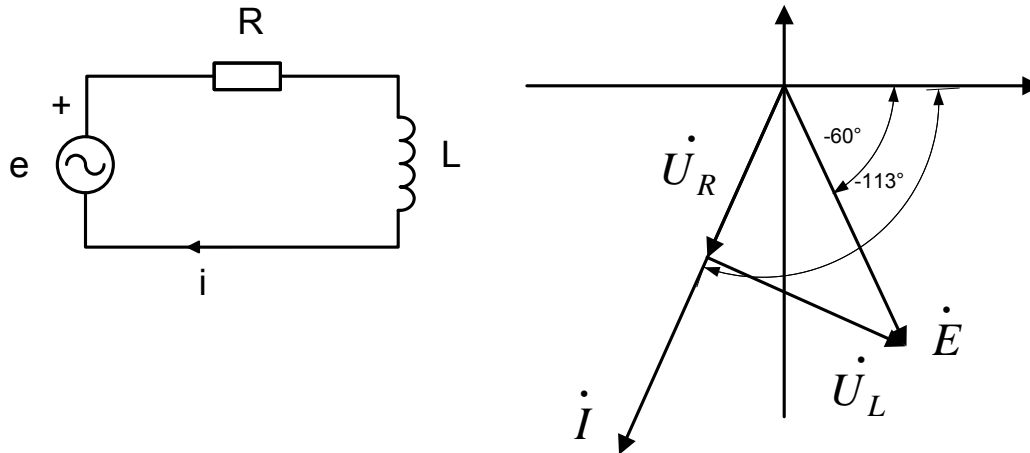
$$w_C(t) = 72,64 \cdot \sin(2\omega t - 30^\circ) + 72,62 \text{ J}$$

$$\dot{S} = \dot{U} \cdot \dot{I}^* = 220 \angle 30^\circ \cdot 207,3 \angle -120^\circ = 45616 \angle -90^\circ \text{ VA}$$

$$\dot{S} = -j45616 \Rightarrow \quad P = 0 ; \quad Q = -45616 \text{ VAR}$$

4. Rješavanje mreža u frekvencijskoj domeni pomoću fazora

4.1. Serijski spoj $R = 60 \, \Omega$ i $L = 0.08 \, \text{H}$ priključen je na EMS $e = 20 \sin(1000t - \frac{\pi}{3}) \, \text{V}$. Treba odrediti struju izvora i sve napone, te ih prikazati pomoću fazorskog dijagrama.



$$\dot{E} = \frac{20}{\sqrt{2}} \angle -60^\circ = 14,14 \angle -60^\circ \, \text{V}$$

$$X_L = \omega L = 10^3 \cdot 80 \cdot 10^{-3} = 80 \, \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{60^2 + 80^2} = 100 \, \Omega$$

$$\varphi = \arctg \frac{X_L}{R} = \arctg \frac{80}{60} = 53,13^\circ$$

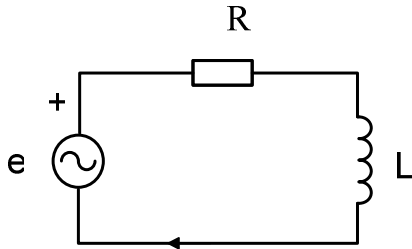
$$\dot{Z} = 100 \angle 53,13^\circ \, \Omega$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{14,14 \angle -60^\circ}{100 \angle 53,13^\circ} = 0,1\sqrt{2} \angle -113,13^\circ \, \text{A}$$

$$\dot{U}_R = \dot{I} \cdot R = 0,1\sqrt{2} \angle -113,13^\circ \cdot 60 = 6\sqrt{2} \angle -113,13^\circ \, \text{V}$$

$$\dot{U}_L = \dot{I} \cdot jX_L = 0,1\sqrt{2} \angle -113,13^\circ \cdot 80 \angle 90^\circ = 8\sqrt{2} \angle -23,13^\circ \, \text{V}$$

4.2. Kroz serijski spoj $R = 8 \Omega$ i $L = 60 \text{ mH}$ teče struja $i = \sqrt{2} \cdot 10 \sin(100t + 30^\circ) \text{ A}$. Treba odrediti sve napone.



$$\dot{I} = 10 \angle 30^\circ \text{ A}$$

$$X_L = \omega L = 100 \cdot 0,06 = 6 \Omega$$

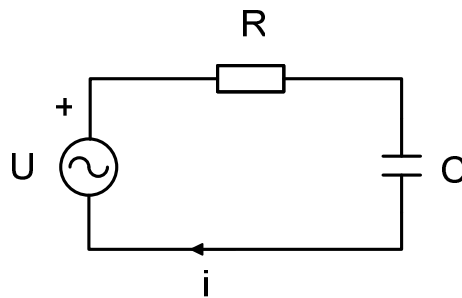
$$\dot{Z} = 10 \angle 36,87^\circ \Omega$$

$$\dot{E} = \dot{I} \cdot \dot{Z} = 100 \angle 66,87^\circ \text{ V}$$

$$\dot{U}_R = \dot{I} \cdot R = 10 \angle 30^\circ \cdot 8 = 80 \angle 30^\circ \text{ V}$$

$$\dot{U}_L = \dot{I} \cdot jX_L = 10 \angle 30^\circ \cdot 6 \angle 90^\circ = 60 \angle 120^\circ \text{ V}$$

4.3. Na serijskom spoju $R = 10 \Omega$ i $C = 31,9 \mu\text{F}$ priključen je napon $U = 127 \text{ V}$. Treba odrediti I , Z i $\text{tg}\varphi$ u slučaju da je: a) $f_1 = 50 \text{ Hz}$ i b) $f_2 = 500 \text{ Hz}$.



a) $f_1 = 50 \text{ Hz}$;

$$X_C = \frac{1}{\omega_C} = \frac{1}{2\pi \cdot 50 \cdot 31,9 \cdot 10^{-6}} = 99,78 \Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 99,78^2} = 100,28 \Omega$$

$$\varphi = \arctg \frac{-X_C}{R} = \arctg \frac{-99,78}{10} = -84,28^\circ$$

$$\dot{Z} = 100,28 \angle -84,28^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{\dot{Z}} = \frac{127}{100,28 \angle -84,28^\circ} = 1,266 \angle 84,28^\circ \text{ A}$$

b) $f_2 = 500 \text{ Hz}$;

$$X_C = \frac{1}{2\pi \cdot 500 \cdot 31,9 \cdot 10^{-6}} = 9,978 \Omega$$

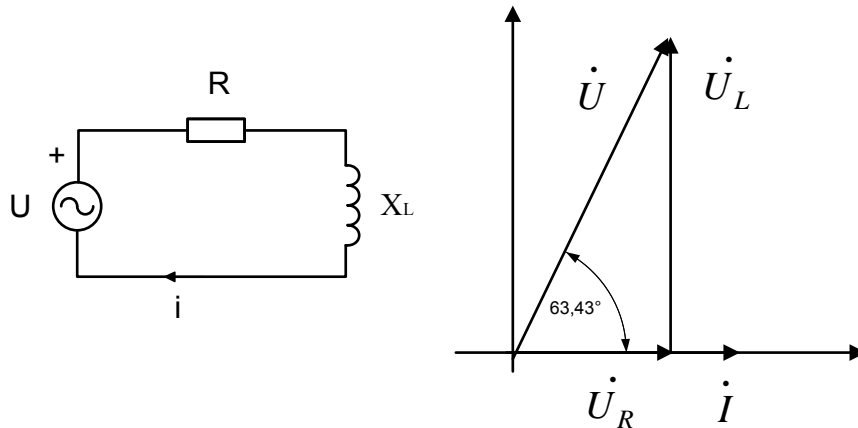
$$Z = \sqrt{10^2 + 9,978^2} = 14,127 \Omega$$

$$\varphi = \arctg \frac{-9,978}{10} = -44,938^\circ$$

$$\dot{Z} = 14,127 \angle -44,938^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{\dot{Z}} = \frac{127}{14,127 \angle -44,938^\circ} = 9 \angle 44,938^\circ \text{ A}$$

4.4. U serijskom spoju $R = 5 \Omega$ i $X_L = 10 \Omega$ poznat je napon $U_R = 10 \text{ V}$; treba odrediti sve ostale napone i struje te ih prikazati fazorski.

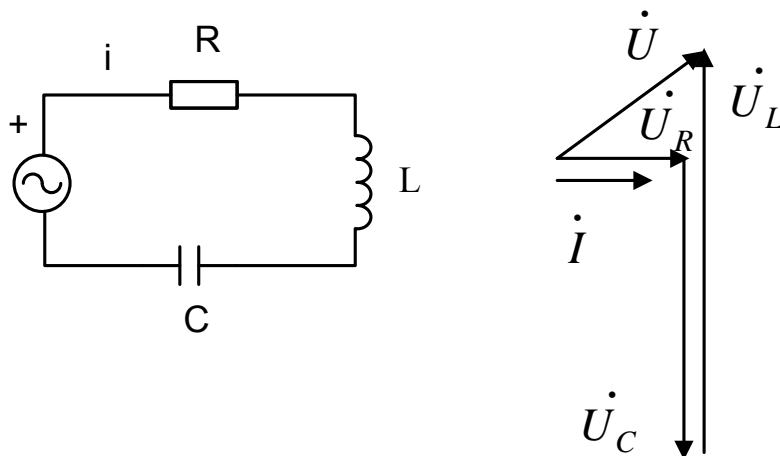


$$\dot{I} = \frac{\dot{U}_R}{R} = \frac{10}{5} = 2 \text{ A}$$

$$\dot{U}_L = \dot{I} \cdot jX_L = 2 \cdot 10 \angle 90^\circ = j20 \text{ V}$$

$$\dot{U} = \dot{U}_R + \dot{U}_L = 10 + j20 = 22,36 \angle 63,43^\circ = 10\sqrt{5} \text{ V}$$

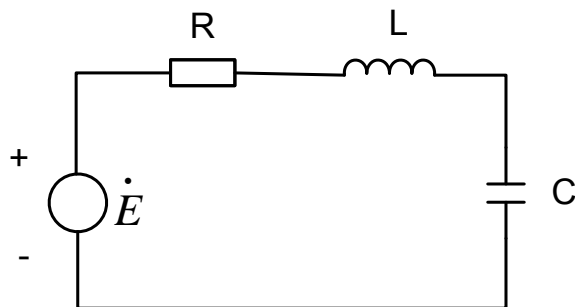
4.5. U serijskom RLC spoju izmjereni su naponi $U_R = 8 \text{ V}$; $U_C = 92 \text{ V}$ i $U_L = 98 \text{ V}$. Treba odrediti ukupni napon U i fazni kut φ spoja te nacrtati fazorski dijagram.



$$U = \sqrt{U_R^2 + (U_L - U_C)^2} = \sqrt{8^2 + (98 - 92)^2} = 10 \text{ V}$$

$$\varphi = \arctg \frac{U_L - U_C}{U_R} = \arctg \frac{98 - 92}{8} = 36,87^\circ \rightarrow \varphi = 36^\circ 52' 11''$$

4.6. U mreži prema slici zadano je $\dot{E} = 120 \angle 0^\circ \text{ V}$, $L = 25 \text{ mH}$, $C = 50 \text{ } \mu\text{H}$. Pri frekvenciji $\omega = 400 \text{ rad/s}$, struja prethodi EMS za kut $63,4^\circ$. Treba odrediti R i napone na svakom elementu.



$$X_L = \omega L = 400 \cdot 25 \cdot 10^{-3} = 10 \text{ } \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \cdot 50 \cdot 10^{-6}} = 50 \text{ } \Omega$$

$$\tan \varphi = -2 = \frac{X_L - X_C}{R}$$

$$R = \frac{X_L - X_C}{-2} = \frac{10 - 50}{-2} = 20 \text{ } \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 44,72 \text{ } \Omega$$

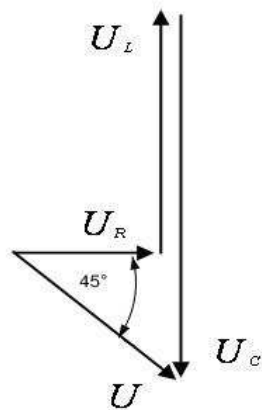
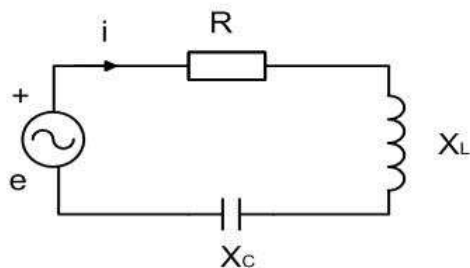
$$I = \frac{E}{Z} = \frac{120}{44,72} = 2,683 \text{ A}$$

$$U_R = I \cdot R = 2,683 \cdot 20 = 53,67 \text{ V}$$

$$U_L = I \cdot X_L = 26,83 \text{ V}$$

$$U_C = I \cdot X_C = 134,16 \text{ V}$$

4.7. Kroz serijski spoj $R = 2 \text{ } \Omega$, $X_L = 4 \text{ } \Omega$ i $X_C = 6 \text{ } \Omega$ teče struja $I = 5 \text{ A}$. Treba odrediti napone na svim elementima i napon na ulazu.



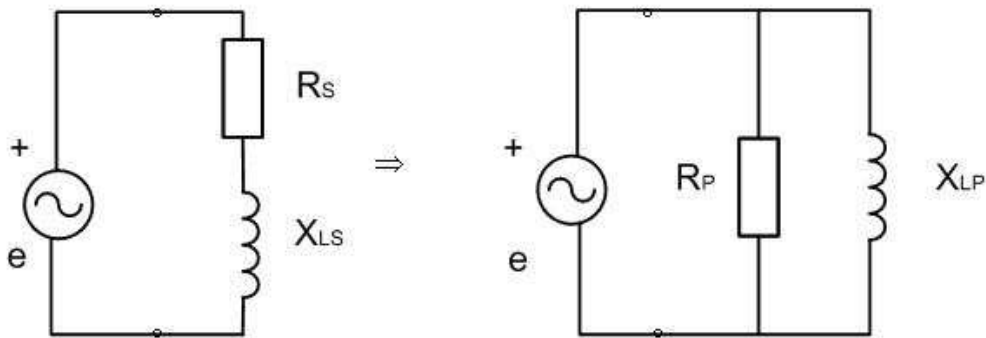
$$U_R = I \cdot R = 5 \cdot 2 = 10 \text{ V}$$

$$U_L = I \cdot X_L = 5 \cdot 4 = 20 \text{ V}$$

$$U_C = I \cdot X_C = 5 \cdot 6 = 30 \text{ V}$$

$$U = \sqrt{U_R^2 + (U_L - U_C)^2} = \sqrt{10^2 + (20 - 30)^2} = 10\sqrt{2} \text{ V}$$

4.8. Treba pretvoriti serijske $R_s = 3,47 \, \Omega$ i $X_{L_s} = 6,56 \, \Omega$ u ekvivalentni paralelni spoj.



$$Z_s = \sqrt{R^2 + X_{L_s}^2} = \sqrt{3,47^2 + 6,56^2} = 7,421 \, \Omega ;$$

$$\varphi_s = \arctg \frac{X_{L_s}}{R} = \arctg \frac{6,56}{3,47} = 62,12^\circ ;$$

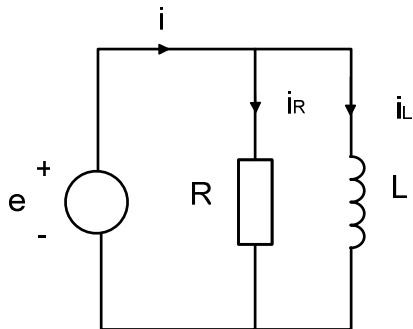
$$\dot{Z} = 7,421 \angle 62,12^\circ \, \Omega ;$$

$$\begin{aligned} \dot{Y}_s &= \frac{1}{\dot{Z}_s} = \frac{1}{7,421 \angle 62,12^\circ} = 0,1347 \angle -62,12^\circ = \\ &= 0,063 - j0,1191 = G - jB_L \end{aligned}$$

$$R_p = \frac{1}{G} = \frac{1}{0,063} = 15,87 \, \Omega$$

$$X_{L_p} = \frac{1}{B_L} = \frac{1}{0,1191} = 8,3955 \, \Omega$$

4.9. Paralelni spoj $R = 60 \, \Omega$ i $L = 0,08 \, H$ priključen je na EMS $e = 20 \sin(1000t - \frac{\pi}{3})$. Odredite sve struje, admitanciju i fazni pomak napona i struja.



$$\dot{E} = \frac{20}{\sqrt{2}} \angle -60^\circ \, V$$

$$i_R = \frac{e}{R} = 0,33 \sin(1000t - \frac{\pi}{3})$$

$$\dot{I}_R = 0,2357 \angle -60^\circ \, A = 0,11785 - j0,204124 \, A$$

$$X_L = \omega L = 10^3 \cdot 0,08 = 80 \, \Omega$$

$$\dot{I}_L = \frac{\dot{E}}{jX_L} = \frac{20 \angle -60^\circ}{\sqrt{2} \cdot 80 \angle 90^\circ} = 0,176777 \angle -150^\circ = -0,153093 - j0,088388 \, A$$

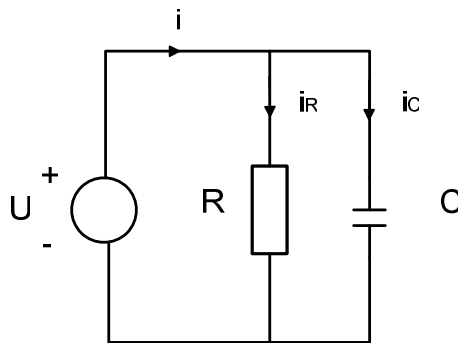
$$\dot{I} = \dot{I}_R + \dot{I}_L = -0,035242 - j0,29251 = 0,29463 \angle -96,87^\circ \, A$$

$$\varphi = \alpha_u - \alpha_i = -60^\circ + 96,87^\circ = 36,87^\circ$$

$$\dot{Y} = \frac{1}{R} - j \frac{1}{X_L} = \frac{1}{60} - j \frac{1}{80} = 0,01\dot{6} - j0,0125 = 0,0208\dot{3} \angle -36,87^\circ S$$

$$\dot{I} = \dot{E} \cdot \dot{Y} = \frac{20}{\sqrt{2}} \angle -60^\circ \cdot 0,0208\dot{3} \angle -36,87^\circ = 0,29463 \angle -96,87^\circ A$$

4.10. Paralelni spoj $R = 10 \Omega$ i $C = 31,9 \mu F$ priključen je na napon $U = 127 V$. Odredite Y , fazni kut φ i sve struje u slučaju kad je: a) $f_1 = 50 Hz$ i b) $f_2 = 500 Hz$.



a) $f_1 = 50 Hz$; $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 50 \cdot 31,9 \cdot 10^{-6}} = 99,78 \approx 100 \Omega$

$$\dot{Y} = \frac{1}{R} + j \frac{1}{X_C} = 0,1 + j0,01 = 0,1005 \angle 5,7106^\circ S$$

$$\varphi = -5,7106^\circ$$

$$\dot{I} = \dot{U} \cdot \dot{Y} = 127 \cdot 0,1005 \angle 5,7106^\circ = 12,7633 \angle 5,7106^\circ = 12,7 + j1,27 A$$

$$\dot{I}_R = 12,7 A \quad ; \quad \dot{I}_C = 1,27 \angle 90^\circ A$$

b) $f_2 = 500 Hz$; $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 500 \cdot 31,9 \cdot 10^{-6}} \approx 10 \Omega$

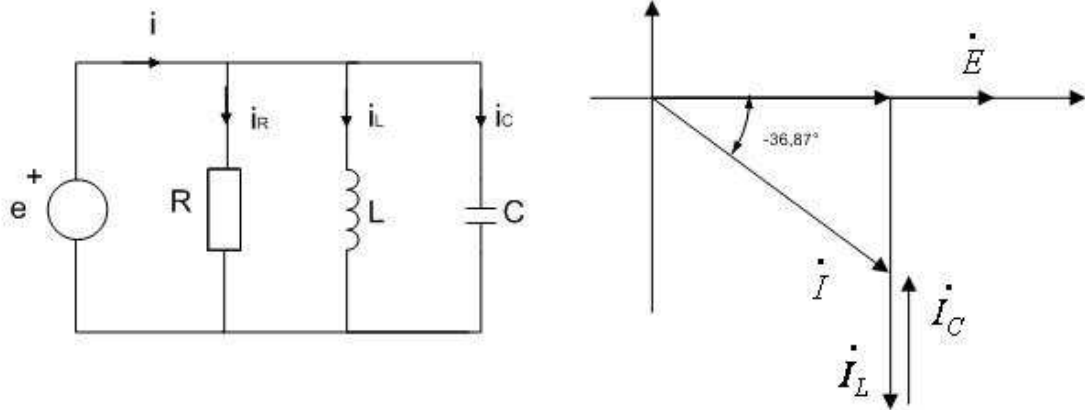
$$\dot{Y} = \frac{1}{R} + j \frac{1}{X_C} = 0,1 + j0,1 = 0,1\sqrt{2} \angle 45^\circ S$$

$$\varphi = -45^\circ$$

$$\dot{I} = \dot{U} \cdot \dot{Y} = 127 \cdot 0,1\sqrt{2} \angle 45^\circ = 12,7 + j12,7 A$$

$$\dot{I}_R = 12,7 A \quad ; \quad \dot{I}_C = j12,7 A$$

4.11. Kroz paralelni RLC spoj teku struje $I_R = 8 \text{ A}$, $I_L = 10 \text{ A}$ i $I_C = 4 \text{ A}$. Odredite struju izvora, zatim L i C ako je $R = 10 \Omega$ i $f = 50 \text{ Hz}$. Nacrtajte fazorski dijagram.



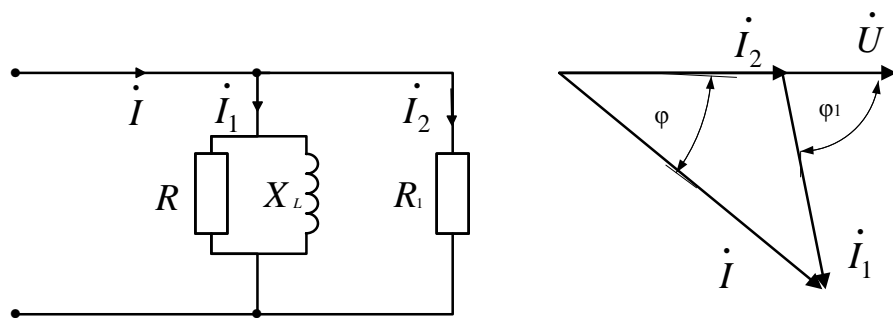
$$\dot{I} = 8 - j6 = 10 \angle -36,87^\circ \text{ A} ; \quad i = 10\sqrt{2} \sin(314t - 36,87^\circ) \text{ A}$$

$$\dot{E} = \dot{I}_R \cdot R = 8 \cdot 10 = 80 \angle 0^\circ \text{ V} ; \quad e = 80\sqrt{2} \sin 314t \text{ V}$$

$$X_L = \frac{\dot{E}}{j\dot{I}_L} = \frac{80 \angle 0^\circ}{1 \angle 90^\circ \cdot 10 \angle -90^\circ} = 8 \Omega ; \quad L = \frac{X_L}{\omega} = \frac{8}{314} = 25,465 \text{ mH}$$

$$X_C = \frac{\dot{E}}{-j\dot{I}_C} = \frac{80 \angle 0^\circ}{1 \angle -90^\circ \cdot 4 \angle 90^\circ} = 20 \Omega ; \quad C = \frac{1}{\omega X_C} = \frac{1}{314 \cdot 20} = 159,15 \mu\text{F}$$

4.12. U spoju prema slici zadan je otpor $R_1 = 4 \Omega$ i izmjerene su efektivne vrijednosti struja $I_1 = 20 \text{ A}$, $I_2 = 15 \text{ A}$ i $I = 30 \text{ A}$; treba odrediti R i X_L .



$$\dot{U} = \dot{I}_2 \cdot 4 = 15 \cdot 4 = 60 \text{ V} \quad Z_1 = \frac{U}{I_1} = \frac{60}{20} = 3 \Omega$$

$$\cos \varphi = \frac{I_2^2 + I^2 - I_1^2}{2 \cdot I_2 \cdot I} = \frac{15^2 + 30^2 - 20^2}{2 \cdot 15 \cdot 30} = 0,805 \Rightarrow \varphi = 36,336^\circ$$

$$\dot{I} = 30 \angle -36,336^\circ = 24,16 - j17,77 \text{ A} \Rightarrow \dot{I}_1 = \dot{I} - \dot{I}_2 = 9,16 - j17,77 = 20 \angle -62,72^\circ \text{ A}$$

$$\dot{Y}_1 = \frac{\dot{I}_1}{\dot{U}} = \frac{20 \angle -62,72^\circ}{60} = 0,1527 - j0,29626 = G - jB \text{ S}$$

$$R = \frac{1}{G} = 6,545 \Omega \quad ; \quad X_L = \frac{1}{B} = 3,375 \Omega$$

4.13. Kroz serijski spoj nepoznatih elemenata teče struja $i = 2,82 \sin(100t - 45^\circ)$ A, pri čemu je napon na vanjskim priključnicama $u = 282 \sin(100t + 45^\circ)$ V. Treba odrediti nepoznate elemente.

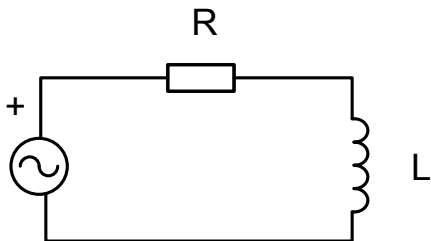
$$\begin{aligned} \dot{I}_m &= 2,82 \angle -45^\circ \text{ A} & \dot{U}_m &= 282 \angle 45^\circ \text{ V} & X_L &= 100 \Omega = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{100}{100} = 1 \text{ H} \\ \dot{Z} &= \frac{\dot{U}_m}{\dot{I}_m} = \frac{282 \angle 45^\circ}{2,82 \angle -45^\circ} = 100 \angle 90^\circ = j100 \Omega & R &= 0 \end{aligned}$$

4.14. Na impedanciju \dot{Z} priključen je napon $\dot{U} = 100 \angle 80^\circ$ V, pri čemu teče struja $\dot{I} = 10 \angle 30^\circ$ A.

Treba odrediti: \dot{Z} , R , X , φ .

$$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{100 \angle 80^\circ}{10 \angle 30^\circ} = 10 \angle 50^\circ = 6,428 + j7,66 \Omega = R + jX$$

4.15. Serijski spoj $R = 6 \Omega$ i $L = 8 \text{ mH}$ priključen je na EMS $e = 100\sqrt{2} \cos(10^3 t - 30^\circ)$ V; treba pomoću fazora odrediti struju i sve napone.



$$X_L = \omega L = 10^3 \cdot 8 \cdot 10^{-3} = 8 \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{6^2 + 8^2} = 10 \Omega$$

$$\varphi = \arctg \frac{X_L}{R} = \arctg \frac{8}{6} = 53,13^\circ$$

$$\dot{Z} = 10 \angle 53,13^\circ \Omega$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{100 \angle -30^\circ}{10 \angle 53,13^\circ} = 10 \angle -83,13^\circ \text{ A}$$

$$U_R = I \cdot R = 10 \cdot 6 = 60 \text{ V}$$

$$U_L = I \cdot X_L = 10 \cdot 8 = 80 \text{ V}$$

5. Električna snaga

5.1. Treba odrediti trokut snage u primjerima 4.1 do 4.7.

$$(4.1) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = 100 \angle 0^\circ \cdot 10 \angle 53,1^\circ = 1000 \angle 53,1^\circ = 600 + j800 \text{ VA} = P + jQ$$

$$(4.2) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = 100 \angle 0^\circ \cdot 10 \angle -53,1^\circ = 1000 \angle -53,1^\circ = 600 - j800 \text{ VA} = P + jQ$$

$$(4.3) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = 100 \angle 53,1^\circ \cdot 10 \angle 0^\circ = 1000 \angle 53,1^\circ = 600 + j800 \text{ VA} = P + jQ$$

$$(4.4) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = 10 \angle 0^\circ \cdot 2,08 \angle 36,87^\circ = 20,8 \angle 36,87^\circ = 16,64 + j12,48 \text{ VA} = P + jQ$$

$$(4.5) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = 100 \angle 0^\circ \cdot 5,6 \angle -26,5^\circ = 560 \angle -26,5^\circ = 501 - j250 \text{ VA} = P + jQ$$

$$(4.6) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = \frac{255}{\sqrt{2}} \angle 45^\circ \cdot \frac{8,5}{\sqrt{2}} \angle -15^\circ = 1083 \angle 30^\circ = 938,6 + j541,88 \text{ VA} = P + jQ$$

$$(4.7) \quad \dot{S} = \dot{E} \cdot \dot{I}^* = \frac{255}{\sqrt{2}} \angle 45^\circ \cdot \frac{8,5}{\sqrt{2}} \angle -15^\circ = 1083 \angle 30^\circ = 938,6 + j541,88 \text{ VA} = P + jQ$$

5.2. Treba odrediti trokut snage u zadacima 4.12 i 4.13.

$$(4.12) \quad \dot{S} = \dot{U} \cdot \dot{I}^* = 60 \cdot 30 \angle 36,336^\circ = 1800 \angle 36,336^\circ = 1450 + j1066 \text{ VA}$$

$$(4.13) \quad \dot{S} = \frac{282}{\sqrt{2}} \angle 45^\circ \cdot \frac{2,82}{\sqrt{2}} \angle 45^\circ = 397,62 \angle 90^\circ = j397,62 \text{ VA}$$

5.3. Treba odrediti trokut snage u zadatku 4.14.

$$\begin{aligned} \dot{U} &= 100 \angle 80^\circ \text{ V} \\ \dot{I} &= 10 \angle 30^\circ \text{ A} \end{aligned} \Rightarrow \dot{S} = \dot{U} \cdot \dot{I}^* = 100 \angle 80^\circ \cdot 10 \angle -30^\circ = 1000 \angle 50^\circ = 642,8 + j766 \text{ VA}$$

5.4. Treba odrediti trokut snage u zadatku 4.15.

$$\begin{aligned} \dot{E} &= 100 \angle -30^\circ \text{ V} \\ \dot{I} &= 10 \angle -83,13^\circ \text{ A} \end{aligned} \Rightarrow \dot{S} = \dot{E} \cdot \dot{I}^* = 100 \angle -30^\circ \cdot 10 \angle 83,13^\circ = 1000 \angle 53,13^\circ = 600 + j800 \text{ VA}$$

5.5. Treba odrediti impedanciju \dot{Z} i struju \dot{I} ako je poznata prividna snaga $S = 4 \text{ kVA}$, faktor snage $\cos \varphi = 0,8$ (*kap.*) i napon $\dot{U} = 220 \angle -30^\circ \text{ V}$.

$$\varphi = \arccos 0,8 = -36,87^\circ$$

$$\dot{S} = 4000 \angle -36,87^\circ \text{ VA} = \dot{U} \cdot \dot{I}^*$$

$$\dot{I}^* = \frac{\dot{S}}{\dot{U}} = \frac{4000 \angle -36,87^\circ}{220 \angle -30^\circ} = 18,18 \angle -6,87^\circ \Rightarrow \dot{I} = 18,18 \angle 6,87^\circ \text{ A}$$

$$\dot{Z} = \frac{\dot{U}}{\dot{I}} = \frac{220 \angle -30^\circ}{18,18 \angle 6,87^\circ} = 12,1 \angle -36,87^\circ \Omega$$

5.6. Treba odrediti impedanciju na kojoj je izmjereno $U = 200 \text{ V}$; $I = 10 \text{ A}$ i $\cos \varphi = 0,8$ (*ind.*).

$$Z = \frac{U}{I} = \frac{200}{10} = 20 \Omega \quad \Rightarrow \quad \dot{Z} = 20 \angle 36,87^\circ \Omega$$

$$\varphi = \arccos 0,8 = 36,87^\circ$$

5.7. U zadatku 5.6 priključen je paralelno impedanciji kapacitet C tako da je izvršena korekcija faktora snage koji iznosi $\cos \varphi' = 0,95$. Treba odrediti jalovu snagu koju je preuzeo kapacitet.

$$\dot{S} = \dot{U} \cdot \dot{I}^* = 200 \angle 0^\circ \cdot 10 \angle 36,87^\circ = 2000 \angle 36,87^\circ = 1600 + j1200 = P + jQ$$

$$S' = \frac{P}{\cos \varphi'} = \frac{1600}{0,95} = 1684,2 \text{ VA}$$

$$Q' = \sqrt{(S')^2 - P^2} = \sqrt{1684,2^2 - 1600^2} = 525,88 \text{ VAR}$$

$$Q_C = Q - Q' = 1200 - 525,88 = 674,11 \text{ VAR}$$

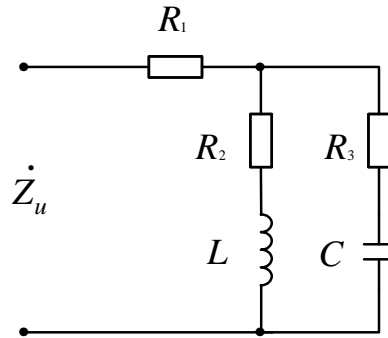
5.8. Na naponski izvor EMS $\dot{E} = 100 \angle 30^\circ$ unutarnje impedancije $\dot{Z}_0 = 6 + j8 \Omega$ treba spojiti takvo trošilo impedancije \dot{Z} da se na njemu troši maksimalna snaga. Treba odrediti parametre trošila i maksimalnu snagu P .

$$\dot{Z}_T = \dot{Z}^* = 6 - j8 \Omega$$

$$P = \frac{E^2}{4R_0} = \frac{100^2}{4 \cdot 6} = 416,6 \text{ W}$$

6. Metode pretvorbe. Fazorski, topografski i locus dijagrami

6.1. Treba odrediti ulaznu impedanciju u shemi na slici, ako je $\omega = 5000 \text{ rad/s}$, $R_1 = 0,5 \Omega$, $R_2 = R_3 = 2 \Omega$, $L = 0,4 \text{ mH}$ i $C = 25 \mu\text{F}$.



$$X_L = \omega L = 5 \cdot 10^3 \cdot 0,4 \cdot 10^{-3} = 2 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{5 \cdot 10^3 \cdot 25 \cdot 10^{-6}} = 8 \Omega$$

$$\dot{Y}_1 = \frac{1}{2 + j2} = \frac{1}{2\sqrt{2} \angle 45^\circ} = 0,3535 \angle -45^\circ = 0,25 - j0,25 \text{ S}$$

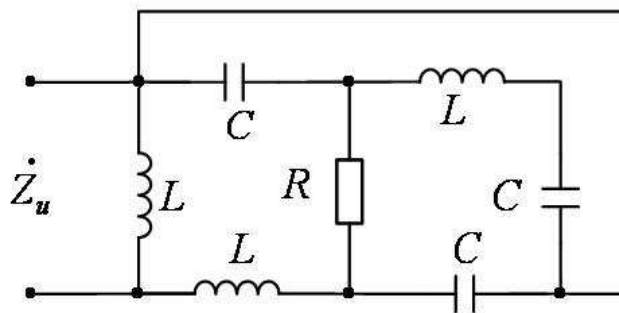
$$\dot{Y}_2 = \frac{1}{2 - j8} = \frac{1}{8,246 \angle -75,96^\circ} = 0,12126 \angle 75,96^\circ = 0,02941 + j0,1176 \text{ S}$$

$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 = 0,27941 - j0,1323 = 0,30917 \angle -25,346^\circ \text{ S}$$

$$\dot{Z}_p = \frac{1}{\dot{Y}_p} = \frac{1}{0,30917 \angle -25,35^\circ} = 3,2344 \angle 25,35^\circ = 2,923 + j1,3846 \Omega$$

$$\dot{Z} = 0,5 + \dot{Z}_p = 3,423 + j1,3846 \Omega$$

6.2. U shemi na slici treba odrediti ulaznu impedanciju ako je: $L = 2 \text{ H}$, $C = 0,33 \mu\text{F}$, $R = 1 \text{ k}\Omega$, $\omega = 1000 \text{ rad/s}$.



$$X_L = \omega L = 10^3 \cdot 2 = 2 \text{ k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^3 \cdot 0,33 \cdot 10^{-6}} = 3,03 \text{ k}\Omega$$

$$\dot{Z}_{12} = j(X_L - X_C) \parallel (-jX_C) = \frac{-j1,03 \cdot (-j3,03)}{-j1,03 - j3,03} = \frac{-3,122}{-j4,06} = -j0,7688 \text{ k}\Omega$$

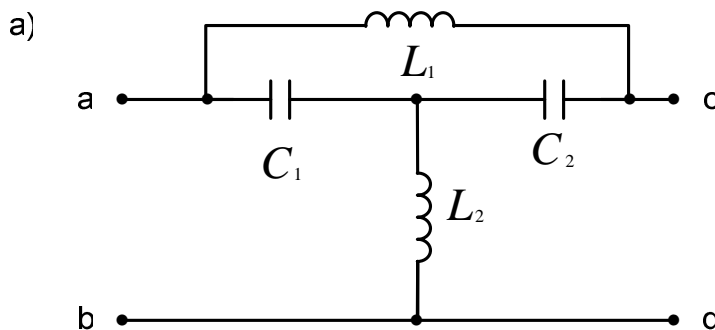
$$\dot{Z}_{12} + R = 1 - j0,7688 = 1,2614 \angle -37,556^\circ \text{ k}\Omega$$

$$\begin{aligned} \dot{Z}_{13} &= \frac{1,2614 \angle -37,556^\circ \cdot (-j3,03)}{1 - j0,7688 - j3,03} = \frac{3,822 \angle -127,55^\circ}{1 - j3,7992} = \frac{3,822 \angle -127,55^\circ}{3,92859 \angle -75,25^\circ} = 0,97299 \angle -52,3^\circ = \\ &= 0,59498 - j0,76988 \text{ k}\Omega \end{aligned}$$

$$\dot{Z}_{13} + jX_L = 0,59498 + j1,2301 = 1,3664 \angle 64,188^\circ \text{ k}\Omega$$

$$\dot{Z}_u = \frac{1,3664 \angle 64,188^\circ}{0,595498 + j1,2301 + j2} = \frac{2,7329 \angle 154,188^\circ}{3,284 \angle 79,56^\circ} = 0,83207 \angle 74,62^\circ \text{ k}\Omega$$

6.3. Treba odrediti \dot{Z}_{ab} u shemi na slici: a) kada su priključnice c-d otvorene; b) kada su priključnice c-d kratko spojene. Zadano: $\omega = 1000 \text{ rad/s}$, $L_1 = L_2 = 50 \text{ mH}$, $C_1 = 10 \mu\text{F}$, $C_2 = 20 \mu\text{F}$.



(serijska rezonancija)

$$X_L = \omega L = 10^3 \cdot 50 \cdot 10^{-3} = 50 \Omega$$

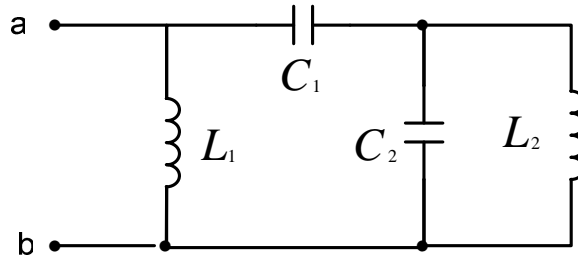
$$X_{C1} = \frac{1}{\omega C_1} = \frac{1}{10^3 \cdot 10 \cdot 10^{-6}} = 100 \Omega$$

$$X_{C2} = \frac{1}{\omega C_2} = \frac{1}{10^3 \cdot 20 \cdot 10^{-6}} = 50 \Omega$$

$$a) \quad \dot{Z}_{a1} = j(X_{L1} - X_{C2}) \parallel (-jX_{C1}) = 0$$

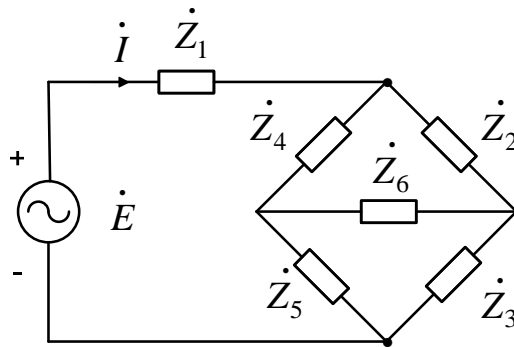
$$\dot{Z}_{ab} = jX_{L2} = j50 \Omega$$

b)



(paralelna rezonancija) b) $\dot{Y}_p = 0 \Rightarrow \dot{Z}_p = \infty \Rightarrow \dot{Z}_{ab} = jX_{L_1} = j50 \Omega$

6.4 U shemi na slici treba odrediti struju \dot{I} , radnu snagu P izvora. Zadano: $\dot{E} = 200 \angle 0^\circ \text{ V}$,
 $\dot{Z}_1 = 113 + j172 \Omega$, $\dot{Z}_2 = 600 - j240 \Omega$, $\dot{Z}_3 = 260 + j200 \Omega$, $\dot{Z}_4 = 200 + j240 \Omega$,
 $\dot{Z}_5 = 140 - j200 \Omega$, $\dot{Z}_6 = 800 \Omega$.



$$\dot{Z}_\Delta = \dot{Z}_3 + \dot{Z}_5 + \dot{Z}_6 = 260 + j200 + 140 - j200 + 800 = 1200 \Omega$$

$$\dot{Z}_a = \frac{\dot{Z}_5 \cdot \dot{Z}_6}{\dot{Z}_\Delta} = \frac{244,13 \angle -55^\circ \cdot 800}{1200} = 162,75 \angle -55^\circ = 93,3 - j133,3 \Omega$$

$$\dot{Z}_b = \frac{\dot{Z}_3 \cdot \dot{Z}_6}{\dot{Z}_\Delta} = \frac{328 \angle 37,57^\circ \cdot 800}{1200} = 218,68 \angle 37,57^\circ = 173,3 + j133,3 \Omega$$

$$\dot{Z}_c = \frac{\dot{Z}_3 \cdot \dot{Z}_5}{\dot{Z}_\Delta} = \frac{328 \angle 37,57^\circ \cdot 244,13 \angle -55^\circ}{1200} = 66,73 \angle -17,43^\circ = 63,66 - j20 \Omega$$

$$\dot{Z}_a + \dot{Z}_4 = 93,3 - j133,3 + 200 + j240 = 293,3 + j106,6 = 312,12 \angle 20^\circ \Omega$$

$$\dot{Z}_b + \dot{Z}_2 = 173,3 + j133,3 + 600 - j240 = 773,3 - j106,6 = 780,6 \angle -7,85^\circ \Omega$$

$$\dot{Z}_p = (\dot{Z}_b + \dot{Z}_2) \parallel (\dot{Z}_a + \dot{Z}_4) = \frac{780,6 \angle -7,85^\circ \cdot 312,12 \angle 20^\circ}{773,3 - j106,6 + 293,3 + j106,6} = \frac{243662 \angle 12,13^\circ}{1066,6} = 228,4 \angle 12,13^\circ = 223,33 + j48 \Omega$$

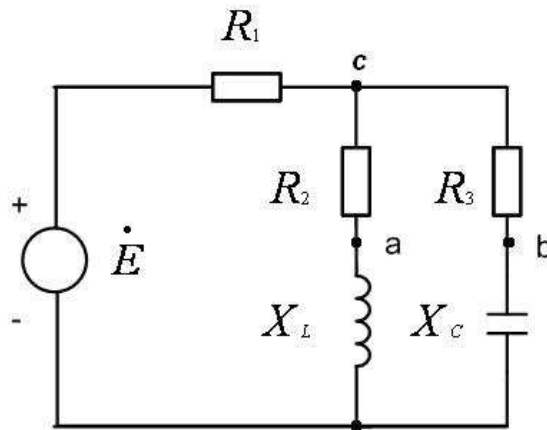
$$\dot{Z} = \dot{Z}_1 + \dot{Z}_p + \dot{Z}_c = 113 + j172 + 223,3 + j48 + 63,66 - j20 = 400 + j200 = 447,2 \angle 26,57^\circ \Omega$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{200}{447,2 \angle 26,57^\circ} = 0,447 \angle -26,57^\circ A$$

$$\dot{S} = \dot{E} \cdot \dot{I}^* = 200 \cdot 0,447 \angle 26,57^\circ = 89,44 \angle 26,57^\circ = 80 + j40 VA \rightarrow P = 80 W$$

6.5. U shemi na slici zadano je

$\dot{U} = 100 \angle 30^\circ V$, $R_1 = 3 \angle 60^\circ \Omega$, $R_2 = 5 \Omega$, $R_3 = 40 \Omega$, $X_C = 30 \Omega$ i $X_L = 2 \Omega$. Treba nacrtati topografski dijagram i odrediti napon \dot{U}_{ab} .



$$\dot{Y}_1 = \frac{1}{5 + j2} = \frac{1}{5,385 \angle 21,8^\circ} = 0,186 \angle -21,8^\circ = 0,1724 - j0,069 S$$

$$\dot{Y}_2 = \frac{1}{40 - j30} = \frac{1}{50 \angle -36,87^\circ} = 0,02 \angle 36,87^\circ = 0,016 - j0,012 S$$

$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 = 0,1884 - j0,05697 = 0,1968 \angle -16,82^\circ S$$

$$\dot{Z}_p = \frac{1}{\dot{Y}_p} = \frac{1}{0,19684 \angle -16,82^\circ} = 5,080 \angle 16,82^\circ = 4,863 + j1,470 \Omega$$

$$\dot{Z} = 3 \angle 60^\circ + \dot{Z}_p = 1,5 + j2,6 + 4,863 + j1,47 = 6,3629 + j4,068 = 7,552 \angle 32,59^\circ \Omega$$

$$\dot{I} = \frac{\dot{U}}{\dot{Z}} = \frac{100 \angle 30^\circ}{7,552 \angle 32,59^\circ} = 13,241 \angle -2,594^\circ = 13,227 - j0,6 A$$

$$\dot{U}_1 = \dot{I} \cdot 3 \angle 60^\circ = 13,241 \angle -2,594^\circ \cdot 3 \angle 60^\circ = 39,722 \angle 57,4^\circ = 21,398 + j33,47 \text{ V}$$

$$\dot{U}_p = \dot{U} - \dot{U}_1 = 86,602 + j50 - 21,398 - j33,47 = 65,205 + j16,53 = 67,27 \angle 14,23^\circ \text{ V}$$

$$\dot{I}_1 = \dot{U}_p \cdot \dot{Y}_1 = 67,27 \angle 14,23^\circ \cdot 0,186 \angle -21,8^\circ = 12,521 \angle -7,572^\circ = 12,402 - j1,6487 \text{ A}$$

$$\dot{I}_2 = \dot{U}_p \cdot \dot{Y}_2 = 67,27 \angle 14,23^\circ \cdot 0,02 \angle 36,87^\circ = 1,3453 \angle 51,098^\circ = 0,84487 + j1,047 \text{ A}$$

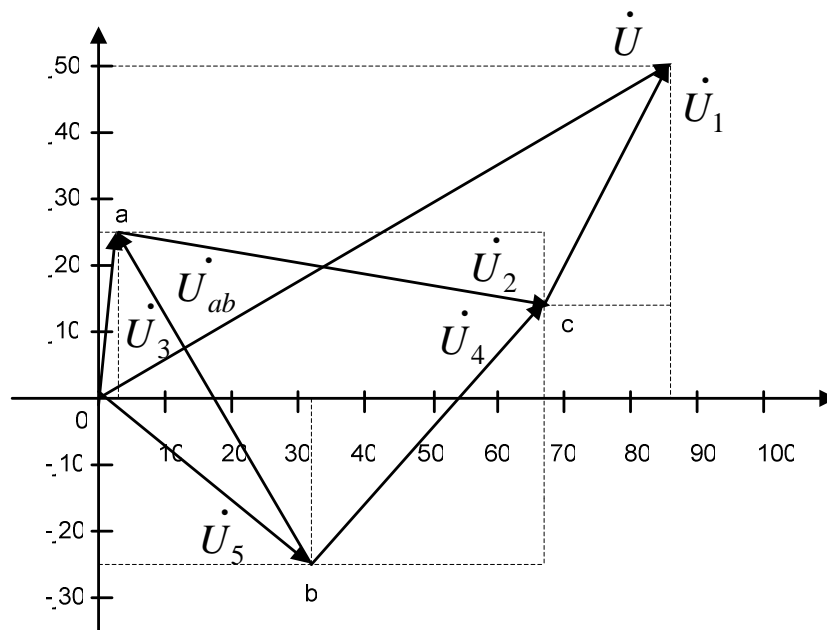
$$\dot{U}_2 = \dot{I}_1 \cdot 5 = 12,512 \angle -7,572^\circ \cdot 5 = 62,56 \angle -7,572^\circ = 62,01 - j8,244 \text{ V}$$

$$\dot{U}_3 = \dot{I}_1 \cdot j2 = 25,024 \angle 82,428^\circ = 3,297 + j24,806 \text{ V}$$

$$\dot{U}_4 = \dot{I}_2 \cdot 40 = 1,3453 \angle 51,098^\circ \cdot 40 = 53,812 \angle 51,098^\circ = 33,793 + j41,878 \text{ V}$$

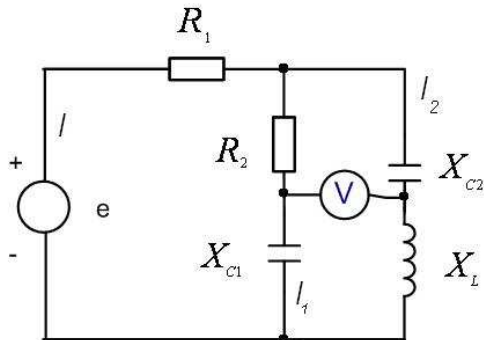
$$\dot{U}_5 = \dot{I}_2 \cdot 30 \angle -90^\circ = 40,359 \angle -38,902^\circ = 31,408 - j25,345 \text{ V}$$

$$\begin{aligned} \dot{U}_{ab} &= \dot{U}_3 - \dot{U}_5 = 3,297 + j24,806 - 31,408 + j25,345 = -28,111 - j50,151 = 57,49 \angle 119,27^\circ \text{ V} \\ &= \dot{U}_4 - \dot{U}_2 \end{aligned}$$



6.6. Koliki napon pokazuje voltmetar u shemi na slici, ako je $e = 30\sqrt{2} \sin(\omega t + 90^\circ)$ V?

Zadano je: $R_1 = 30 \Omega$, $R_2 = 10 \Omega$, $X_{C1} = 60 \Omega$, $X_{C2} = 20 \Omega$ i $X_L = 20 \Omega$.



$$\dot{Z}_2 = -j20 + j20 = 0 \Omega$$

$$\dot{U}_1 + \dot{U}_5 = 0 \Rightarrow \dot{I}_1 = 0; \dot{I}_2 = \dot{I}$$

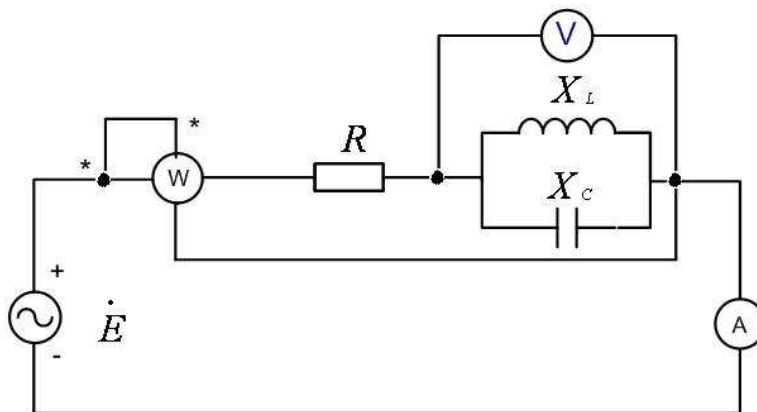
$$\dot{U}_4 = \dot{I} \cdot (-j20) = 1 \angle 90^\circ \cdot 20 \angle -90^\circ = 20 \angle 0^\circ \text{ V}$$

$$\dot{U}_5 = \dot{I} \cdot j20 = 1 \angle 90^\circ \cdot 20 \angle 90^\circ = 20 \angle 180^\circ \text{ V}$$

$$U_V = U_5 = 20 \text{ V}$$

6.7. Koje vrijednosti pokazuju instrumenti na slici? Zadano je $\dot{E} = 100 \angle 30^\circ \text{ V}$, $R = 5 \Omega$,

$X_L = 5 \Omega$ i $X_C = 2,5 \Omega$.



$$\dot{Z}_p = \frac{j5(-j2,5)}{j5 - j2,5} = \frac{12,5}{j2,5} = -j5 \Omega$$

$$\dot{Z} = 5 - j5 = 5\sqrt{2} \angle -45^\circ \Omega$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{100 \angle 30^\circ}{5\sqrt{2} \angle -45^\circ} = 14,14 \angle 75^\circ \text{ A}$$

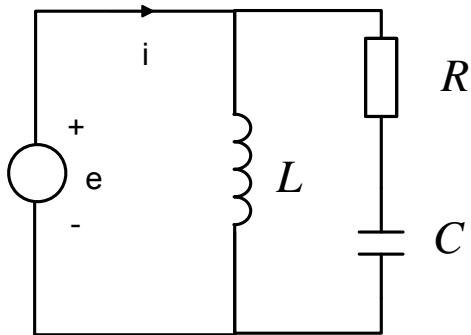
$$\dot{U}_V = \dot{I} \cdot \dot{Z}_p = 14,14 \angle 75^\circ \cdot 5 \angle -90^\circ = 70,71 \angle -15^\circ \text{ V}$$

$$P_W = U \cdot I \cdot \cos \varphi = I^2 \cdot R = 14,14^2 \cdot 5 = 1000 \text{ W}$$

6.8. Treba odrediti induktivitet L u shemi na slici, pod uvjetom da EMS

$$e = \sqrt{2} \cdot 220 \sin(\omega t + 60^\circ) \text{ V prethodi struji izvora } i \text{ za } 30^\circ.$$

$$\text{Zadano: } f = 50 \text{ Hz, } R = 150 \, \Omega, \, C = 33 \, \mu\text{F}.$$



$$\omega = 2\pi f = 100\pi = 314 \text{ rad/s}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \cdot 33 \cdot 10^{-6}} = 96,46 \, \Omega$$

$$\dot{E} = 220 \angle 60^\circ \text{ V}$$

$$\dot{Y}_2 = \frac{1}{R - jX_C} = \frac{1}{150 - j96,46} = \frac{1}{178,34 \angle -32,74^\circ} = 5,607 \cdot 10^{-3} \angle 32,74^\circ$$

$$\dot{Y}_2 = (4,716 + j3,03) \cdot 10^{-3} \text{ S}$$

$$\dot{Y}_1 = \frac{1}{jX_L} = -j \frac{1}{X_L}$$

$$\dot{Y} = \dot{Y}_1 + \dot{Y}_2 = 4,716 \cdot 10^{-3} + j(3,03 \cdot 10^{-3} - \frac{1}{X_L})$$

$$\dot{I} = \dot{E} \cdot \dot{Y} = E \cdot Y \angle 60^\circ - 30^\circ = E \cdot Y \angle 30^\circ$$

$$\Psi = -30^\circ = \arctg \frac{3,03 \cdot 10^{-3} - \frac{1}{X_L}}{4,716 \cdot 10^{-3}} / \text{tg}$$

$$\text{tg}(-30^\circ) = \frac{3,03 \cdot 10^{-3} - \frac{1}{X_L}}{4,716 \cdot 10^{-3}} = -0,57735 / 4,716 \cdot 10^{-3}$$

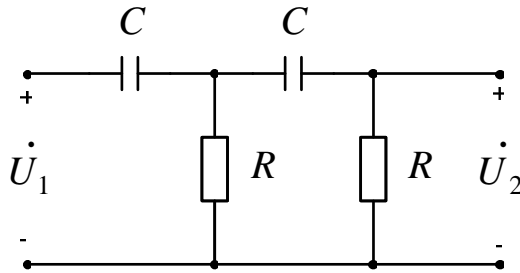
$$3,03 \cdot 10^{-3} - \frac{1}{X_L} = -2,723 \cdot 10^{-3}$$

$$\frac{1}{X_L} = 3,03 \cdot 10^{-3} + 2,723 \cdot 10^{-3} = 5,7558 \cdot 10^{-3}$$

$$X_L = 173,73 = \omega L$$

$$L = \frac{173,73}{314} = 0,553 \text{ H}$$

6.9. U shemi na slici treba odrediti C tako da, uz poznati R , napon \dot{U}_2 prethodi naponu \dot{U}_1 za 90° .



$$\dot{U}_1 = (R - jX_C) \cdot \dot{I}_1 - R \cdot \dot{I}_2$$

$$0 = -R \cdot \dot{I}_1 + (2R - jX_C) \cdot \dot{I}_2 \Rightarrow \dot{I}_1 = \frac{2R - jX_C}{R} \cdot \dot{I}_2$$

$$\dot{U}_1 = (R - jX_C) \cdot \frac{2R - jX_C}{R} \cdot \dot{I}_2 - R \cdot \dot{I}_2 = \frac{2R^2 - jRX_C - j2RX_C - X_C^2 - R^2}{R} \cdot \dot{I}_2$$

$$\dot{U}_1 = \frac{R^2 - j3RX_C - X_C^2}{R} \cdot \dot{I}_2 = \left[\frac{R^2 - X_C^2}{R} - j3X_C \right] \cdot \frac{\dot{U}_2}{R}$$

$$\dot{U}_2 = \dot{I}_2 \cdot R \Rightarrow \dot{I}_2 = \frac{\dot{U}_2}{R}$$

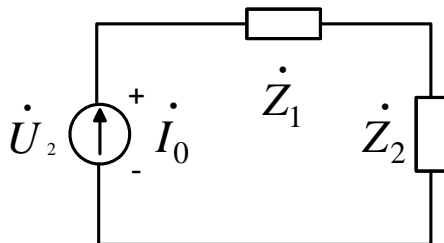
$$\frac{\dot{U}_1}{\dot{U}_2} = \frac{R^2 - X_C^2}{R^2} - j \frac{3X_C}{R}$$

$$\frac{R^2 - X_C^2}{R^2} = 0 / R^2$$

$$R^2 - X_C^2 = 0 \Rightarrow R^2 = X_C^2$$

$$R = X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega R}$$

6.10. U shemi prema slici treba odrediti trokut snage na impedancijama \dot{Z}_1 , \dot{Z}_2 i na strujnom izvoru, ako je zadano: $\dot{Z}_1 = 6 \angle 60^\circ \Omega$, $\dot{Z}_2 = 8 \angle 75^\circ \Omega$ i $\dot{I}_1 = 3 \angle 30^\circ A$.



$$\dot{U}_1 = \dot{I}_0 \cdot \dot{Z}_1 = 3 \angle 30^\circ \cdot 6 \angle 60^\circ = 18 \angle 90^\circ V$$

$$\dot{U}_2 = \dot{I}_0 \cdot \dot{Z}_2 = 3 \angle 30^\circ \cdot 8 \angle 75^\circ = 24 \angle 105^\circ V$$

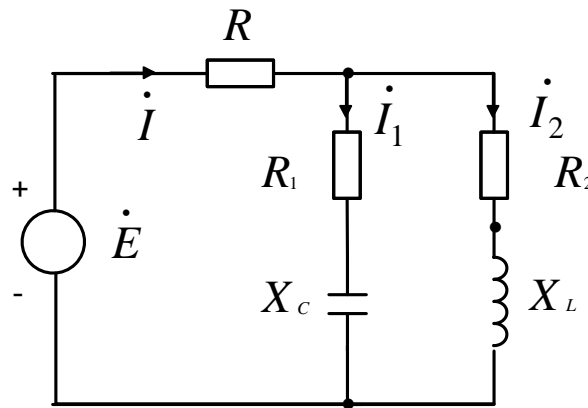
$$\dot{S}_1 = \dot{U}_1 \cdot \dot{I}_0^* = 18 \angle 90^\circ \cdot 3 \angle -30^\circ = 54 \angle 60^\circ = 27 + j46,77 VA$$

$$\dot{S}_2 = \dot{U}_2 \cdot \dot{I}_0^* = 24 \angle 105^\circ \cdot 3 \angle -30^\circ = 72 \angle 75^\circ = 18,63 + j69,55 VA$$

$$\dot{S}_0 = 45,63 + j116,32 = 125 \angle 68,6^\circ VA$$

6.11. U shemi na slici treba odrediti trenutne vrijednosti svih struja i trokut snage izvora.

Zadano: $R = 2,68 \Omega$, $R_1 = 10 \Omega$, $R_2 = 5\sqrt{3} \Omega$, $X_C = 10 \Omega$, $X_L = 5 \Omega$, $\dot{E} = 25 \angle 30^\circ V$, $f = 50 Hz$.



$$\dot{Y}_1 = \frac{1}{10 - j10} = \frac{1}{14,14 \angle -45^\circ} = 0,07071 \angle 45^\circ = 0,05 + j0,05 S$$

$$\dot{Y}_2 = \frac{1}{5\sqrt{3} + j5} = \frac{1}{10 \angle 30^\circ} = 0,1 \angle -30^\circ = 0,0866 - j0,05 S$$

$$\dot{Y}_p = 0,1366 \angle 0^\circ S$$

$$\dot{Z}_p = 7,32 \Omega \Rightarrow \dot{Z} = R + \dot{Z}_p = 10 \Omega$$

$$\dot{I} = \frac{25 \angle 30^\circ}{10} = 2,5 \angle 30^\circ \text{ A} \Rightarrow \dot{U}_R = \dot{I} \cdot R = 2,5 \angle 30^\circ \cdot 2,68 = 6,7 \angle 30^\circ \text{ V}$$

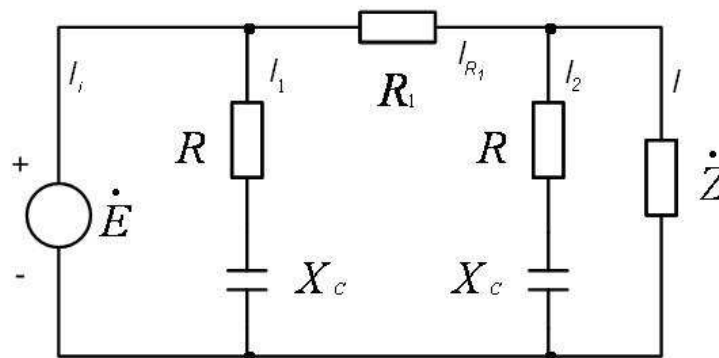
$$\dot{U}_p = \dot{E} - \dot{U}_R = 25 \angle 30^\circ - 6,7 \angle 30^\circ = 18,3 \angle 30^\circ \text{ V}$$

$$\dot{I}_1 = \dot{U}_p \cdot \dot{Y}_1 = 18,3 \angle 30^\circ \cdot 0,07071 \angle 45^\circ = 1,294 \angle 75^\circ \text{ A}$$

$$\dot{I}_2 = \dot{U}_p \cdot \dot{Y}_2 = 18,3 \angle 30^\circ \cdot 0,1 \angle -30^\circ = 1,83 \text{ A}$$

$$\dot{S} = \dot{E} \cdot \dot{I}^* = 25 \angle 30^\circ \cdot 2,5 \angle -30^\circ = 62,5 \text{ VA}$$

6.12. U zadanoj shemi na slici treba odrediti impedanciju \dot{Z} tako da kroz nju teče struja $\dot{I} = 1 \angle 0^\circ \text{ A}$. Zadano: $\dot{E} = 10 \angle 30^\circ \text{ V}$, $R = X_C = 5 \Omega$, $R_l = 10 \Omega$.



$$\dot{U}_p = \dot{I} \cdot \dot{Z} = 1 \angle 0^\circ \cdot \dot{Z} = \dot{Z}$$

$$\dot{Z}_2 = R - jX_C = 5 - j5 = 5\sqrt{2} \angle -45^\circ \Omega$$

$$\dot{I}_2 = \frac{\dot{U}_p}{\dot{Z}_2} = \frac{\dot{Z}}{5\sqrt{2} \angle -45^\circ}$$

$$\dot{I}_{Rl} = \dot{I}_2 + \dot{I} = \frac{\dot{Z}}{5\sqrt{2} \angle -45^\circ} + 1$$

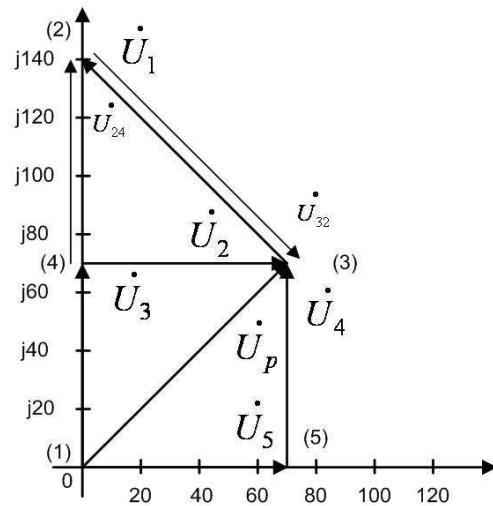
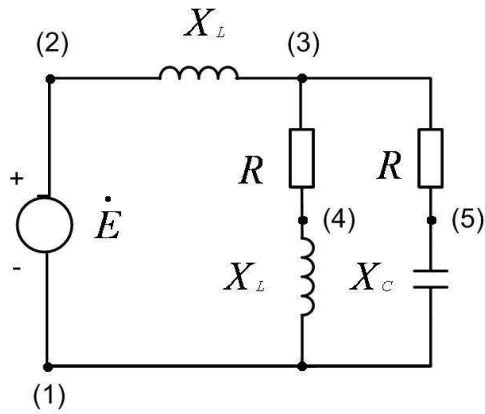
$$\dot{U}_{Rl} = R_l \cdot \dot{I}_{Rl} = 10 \left[\frac{\dot{Z}}{5\sqrt{2} \angle -45^\circ} + 1 \right] = \dot{Z} \sqrt{2} \angle 45^\circ + 10$$

$$\dot{E} = \dot{U}_{Rl} + \dot{U}_p = \dot{Z} \sqrt{2} \angle 45^\circ + 10 + \dot{Z} = 10 \angle 30^\circ$$

$$\dot{Z}(\sqrt{2} \angle 45^\circ + 1) = 10 \angle 30^\circ - 10$$

$$\dot{Z} = \frac{8,66 + j5 - 10}{1 + j1 + 1} = \frac{-1,3397 + j5}{2 + j} = \frac{5,1764 \angle 105^\circ}{2,2361 \angle 26,565^\circ} = 2,315 \angle 78,43^\circ \Omega$$

6.13. U shemi na slici zadano je: $\dot{E} = 100\sqrt{2} \angle 90^\circ \text{ V}$, $R = X_L = X_C = 10 \Omega$. Treba nacrtati topografski dijagram i odrediti napone \dot{U}_{32} i \dot{U}_{24} .



$$\dot{Y}_1 = \frac{1}{10 + j10} = \frac{1}{10\sqrt{2} \angle 45^\circ} = 0,07071 \angle -45^\circ = 0,05 - j0,05 \text{ S}$$

$$\dot{Y}_2 = \frac{1}{10 - j10} = \frac{1}{10\sqrt{2} \angle -45^\circ} = 0,07071 \angle 45^\circ = 0,05 + j0,05 \text{ S}$$

$$\dot{Y}_p = 0,1 \text{ S} \Rightarrow \dot{Z}_p = 10 \Omega$$

$$\dot{Z} = \dot{Z}_p + jX_L = 10 + j10 = 10\sqrt{2} \angle 45^\circ \Omega$$

$$\dot{I} = \frac{\sqrt{2} \cdot 100 \angle 90^\circ}{10\sqrt{2} \angle 45^\circ} = 10 \angle 45^\circ \text{ A}$$

$$\dot{U}_1 = \dot{I} \cdot jX_L = 10 \angle 45^\circ \cdot 10 \angle 90^\circ = 100 \angle 135^\circ = -70,71 + j70,71 \text{ V}$$

$$\dot{U}_p = \dot{E} - \dot{U}_1 = j100\sqrt{2} + 70,71 - j70,71 = 70,71 + j70,71 = 100 \angle 45^\circ \text{ V}$$

$$\dot{I}_1 = \dot{U}_p \cdot \dot{Y}_1 = 100 \angle 45^\circ \cdot 0,07071 \angle -45^\circ = 7,071 \text{ A}$$

$$\dot{I}_2 = \dot{U}_p \cdot \dot{Y}_2 = 100 \angle 45^\circ \cdot 0,07071 \angle 45^\circ = j7,071 \text{ A}$$

$$\dot{U}_3 = \dot{I}_1 \cdot jX_L = 7,071 \cdot 10 \angle 90^\circ = j70,71 \text{ V}$$

$$\dot{U}_2 = \dot{I}_1 \cdot R = 7,071 \cdot 10 = 70,71 \text{ V}$$

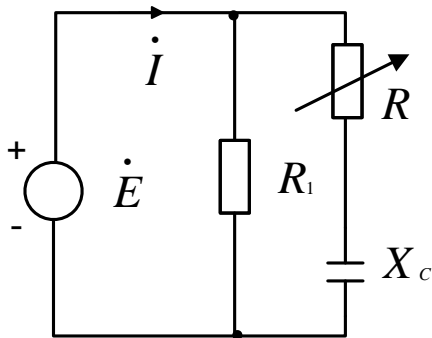
$$\dot{U}_4 = \dot{I}_2 \cdot R = j7,071 \cdot 10 = j70,71 \text{ V}$$

$$\dot{U}_5 = \dot{I}_2 \cdot (-jX_C) = j7,071 \cdot (-j)10 = j70,71 \text{ V}$$

$$\dot{U}_{32} = -\dot{U}_1 = 100 \angle -45^\circ \text{ V}$$

$$\dot{U}_{24} = 70,71 \angle 90^\circ \text{ V}$$

6.14. U shemi prema slici zadano je: $R_1 = 10 \Omega$, $X_C = 4 \Omega$, a otpor R je promjenjiv. Treba odrediti takav otpor R da je fazni pomak između struje i napona 45° .



$$\dot{I} = \frac{\dot{E}}{R_1} + \frac{\dot{E}}{R - jX_C} = \dot{E} \cdot \left(\frac{1}{R_1} + \frac{1}{R - jX_C} \right)$$

$$\frac{\dot{I}}{\dot{E}} = \frac{1}{R_1} + \frac{1}{R - jX_C} \cdot \frac{R + jX_C}{R + jX_C} = \frac{1}{R_1} + \frac{R + jX_C}{R^2 + X_C^2}$$

$$\dot{Y} = \frac{1}{R_1} + \frac{R}{R^2 + X_C^2} + j \cdot \frac{X_C}{R^2 + X_C^2}$$

$$\frac{1}{R_1} + \frac{R}{R^2 + X_C^2} = \frac{X_C}{R^2 + X_C^2} \quad / \cdot (R^2 + X_C^2) \cdot R_1$$

$$R^2 + X_C^2 + R \cdot R_1 = X_C \cdot R_1$$

$$R^2 + R_1 \cdot R + X_C \cdot (X_C - R_1) = 0$$

$$R^2 + 10R - 24 = 0$$

$$R' = 2 \Omega$$

$$R'' = -12 \Omega \quad - \text{nije rješenje}$$

6.15. Na tri trošila, priključena paralelno na mrežu, izmjereni su sljedeći podaci:

a) $S_1 = 1500 \text{ VA}$, $\cos \varphi_1 = 0.8 \text{ (ind)}$, b) $S_2 = 2000 \text{ VA}$, $\cos \varphi_2 = 0.6 \text{ (kap)}$,

c) $S_3 = 2500 \text{ VA}$, $\cos \varphi_3 = 0.9 \text{ (ind)}$. Treba izračunati trokut ukupne snage, kao i faktor snage.

a)

$$\varphi_1 = 36,87^\circ ; P_1 = S_1 \cos \varphi_1 = 1500 \cdot 0,8 = 1200 \text{ W}$$

$$\sin \varphi_1 = 0,6 ; Q_1 = S_1 \sin \varphi_1 = 1500 \cdot 0,6 = 500 \text{ VAR}$$

b)

$$\varphi_2 = -53,13^\circ ; P_2 = S_2 \cos \varphi_2 = 2000 \cdot 0,6 = 1200 \text{ W}$$

$$\sin \varphi_2 = -0,8 ; Q_2 = S_2 \sin \varphi_2 = 2000 \cdot (-0,8) = -1600 \text{ VAR}$$

c)

$$\varphi_3 = 25,84^\circ ; P_3 = S_3 \cos \varphi_3 = 2500 \cdot 0,9 = 2250 \text{ W}$$

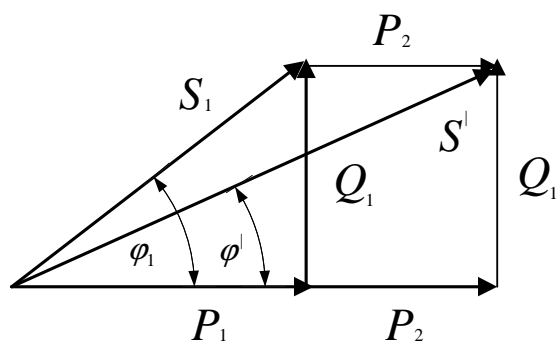
$$\sin \varphi_3 = 0,4359 ; Q_3 = S_3 \sin \varphi_3 = 2500 \cdot 0,4359 = 1089,7 \text{ VAR}$$

$$P = P_1 + P_2 + P_3 = 4650 \text{ W}$$

$$Q = Q_1 + Q_2 + Q_3 = 389,72 \text{ VAR}$$

$$\dot{S} = P + jQ = 4666,3 \angle 4,79^\circ \text{ VA} ; \cos \varphi = 0,9965$$

6.16. U nekom pogonu izmjerena je prividna snaga $S_1 = 10 \text{ kVA}$ i $\cos \varphi_1 = 0,8$ (*ind*). Nakon priključenja električnih grijalica ($\cos \varphi_2 = 1$), novi faktor snage iznosi $\cos \varphi' = 0,9$. Treba odrediti radnu snagu priključenih grijalica.



$$\varphi_1 = 36,87^\circ$$

$$\sin \varphi_1 = 0,6$$

$$P_1 = S_1 \cdot \cos \varphi_1 = 10^4 \cdot 0,8 = 8 \text{ kW}$$

$$Q_1 = S_1 \cdot \sin \varphi_1 = 10^4 \cdot 0,6 = 6 \text{ kVAR}$$

$$\cos \varphi' = 0,9 \Rightarrow \varphi' = 25,84^\circ$$

$$\tan \varphi' = \frac{Q_1}{P}$$

$$P = \frac{Q_1}{\tan \varphi'} = \frac{6 \cdot 10^3}{\tan 25,84^\circ} = 12388,45 \text{ W} = P_1 + P_2$$

$$P_2 = 12388,45 - 8 \cdot 10^3 = 4388,45 \text{ W}$$

6.17. U pogonu je izmjerena prividna snaga $S = 10 \text{ kVA}$, a nakon priključenja sinkronog motora prividne snage $S_1 = 1,5 \text{ kVA}$ i $\cos \varphi_1 = 0,65$ (*kap*), izmjeren je faktor snage $\cos \varphi' = 0,9$ (*ind*). Treba odrediti faktor snage ($\cos \varphi$) prethodnog opterećenja $S = 10 \text{ kVA}$.

$$\cos \varphi_1 = 0,65 \Rightarrow \varphi_1 = 49,458^\circ$$

$$\cos \varphi' = 0,9 \Rightarrow \varphi' = 25,842^\circ$$

$$\varphi_1 + \varphi' = 75,3^\circ$$

$$S^2 = S_1^2 + S'^2 - 2S_1S'\cos 75,3^\circ$$

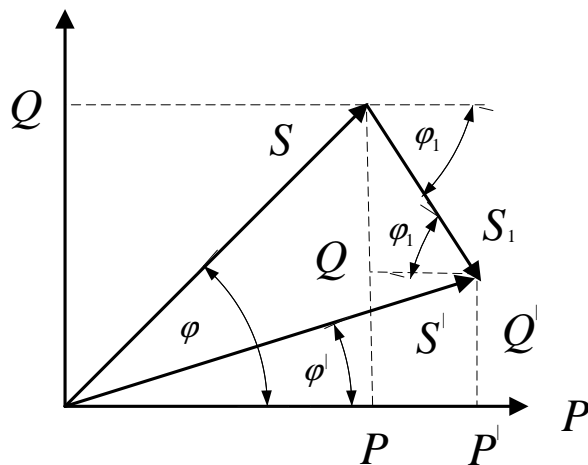
$$10^8 = 1500^2 + S'^2 - 2 \cdot 1500 \cdot S' \cos 75,3^\circ$$

$$S'^2 - 761,257 \cdot S' - 97,75 \cdot 10^6 = 0$$

$$S' = 10274,8 \text{ VA}$$

$$P_1 = S_1 \cos \varphi_1 = 1500 \cdot 0,65 = 975 \text{ W}$$

$$Q_1 = S_1 \cdot \sin \varphi_1 = 1500 \cdot (-0,76) = -1139,9 \text{ VAR}$$



$$P' = S' \cdot \cos \varphi' = 10274,8 \cdot 0,9 = 9247 \text{ W}$$

$$P = P' - P_1 = 9247,3 - 975 = 8272,3 \text{ W}$$

$$Q' = S' \sin \varphi' = 10274,8 \cdot 0,43588 = 4478,68 \text{ VAR}$$

$$Q = Q' - Q_1 = 4478,68 + 1139,9 = 5618,58 \text{ VAR}$$

$$\dot{S} = P + jQ = 10^4 \angle 34,184^\circ$$

$$\cos \varphi = 0,8272$$

6.18. Transformator prividne snage $S = 30 \text{ kVA}$ opterećen je trošilom prividne snage 30 kVA i $\cos \varphi = 0,75$ (*ind.*). Nakon kompenzacije pomoću kondenzatorske baterije, faktor snage iznosi $\cos \varphi' = 0,9$ (*ind.*).

a) Treba odrediti jalovu snagu koju je preuzela kondenzatorska baterija.

b) Kojom se dodatnom snagom faktora snage $\cos \varphi = 1$ smije opteretiti transformator nakon kompenzacije?

$$\varphi = 41,41^\circ \Rightarrow \sin \varphi = 0,6614$$

$$\varphi' = 25,84^\circ \Rightarrow \tan \varphi' = 0,4843$$

$$Q = S \cdot \sin \varphi = 30 \cdot 10^3 \cdot 0,6614 = 19843 \text{ VAR}$$

$$P = S \cdot \cos \varphi = 30 \cdot 10^3 \cdot 0,75 = 22500 \text{ W}$$

a)

$$\tan \varphi' = \frac{Q'}{P} \Rightarrow Q' = P \cdot \tan \varphi' = 22500 \cdot 0,4843 = 10897,2 \text{ VAR}$$

$$Q_c = Q - Q' = 19843 - 10897 = 8946 \text{ VAR}$$

b)

$$P' = \sqrt{S^2 - Q'^2} = \sqrt{(30 \cdot 10^3)^2 - 10897,2^2} = 27950 \text{ W}$$

$$P' - P = 27950 - 22500 = 5450,85 \text{ W}$$

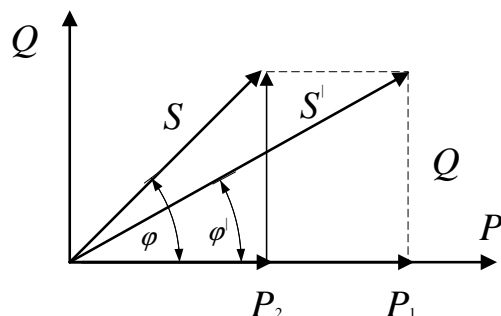
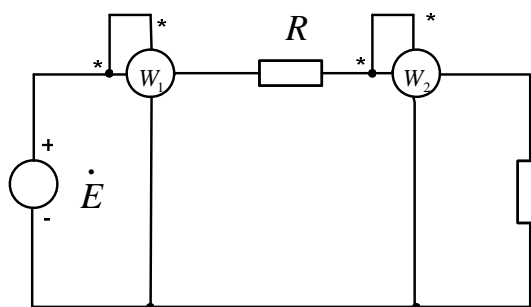
6.19. U shemi na slici vatmetri pokazuju: $P_1 = 208 \text{ W}$, $P_2 = 160 \text{ W}$. Ako je $R = 12 \Omega$ i $E = 120 \text{ V}$, treba odrediti kut impedancije \dot{Z} .

$$P_1 - P_2 = 208 - 160 = 48 \text{ W} = I^2 \cdot R = I^2 \cdot 12$$

$$I^2 = \frac{48}{12} = 4 \Rightarrow I = 2 \text{ A}$$

$$P_1 = E \cdot I \cdot \cos \varphi' \Rightarrow \cos \varphi' = \frac{P_1}{EI} = \frac{208}{120 \cdot 2} = 0,86$$

$$\varphi' = 29,926^\circ \rightarrow \tan \varphi' = 0,5756$$

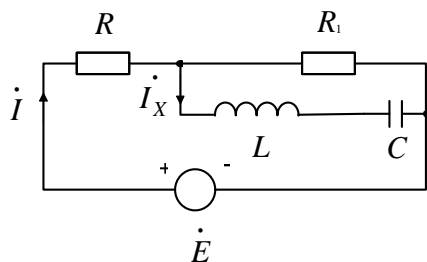


$$\frac{Q}{P_1} = \tan \varphi' \rightarrow Q = P_1 \cdot \tan \varphi' = 208 \cdot 0,5756 = 119,733 \text{ VAR}$$

$$\tan \varphi = \frac{Q}{P_2} = \frac{119,733}{160} = 0,74833$$

$$\varphi = 36,8^\circ$$

6.20. U mreži na slici treba odrediti: otpor R , napone U_L i U_C , ako je zadano: $\dot{E} = 100 \angle 0^\circ \text{ V}$, $\dot{I} = I \angle -18^\circ \text{ A}$, $\omega = 400 \text{ rad/s}$, $R_l = 30 \Omega$, $L = 225 \text{ mH}$ i $C = 50 \mu\text{F}$.



$$X_L = \omega L = 400 \cdot 0,225 = 90 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \cdot 50 \cdot 10^{-6}} = 50 \Omega$$

$$\dot{Y}_1 = \frac{1}{R_l} = \frac{1}{30}$$

$$\dot{Y}_2 = \frac{1}{j(X_L - X_C)} = \frac{-j}{90 - 50} = -0,025j$$

$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 = \frac{1}{30} - j0,025 =$$

$$= 0,041\bar{6} \angle -36,87^\circ$$

$$\dot{Z}_p = \frac{1}{0,041\bar{6} \angle -36,87^\circ} = 24 \angle 36,87^\circ =$$

$$= 19,2 + j14,4 \Omega$$

$$\dot{Z} = \dot{Z}_p + R = 19,2 + R + j14,4$$

$$\tan \varphi = \frac{14,4}{19,2 + R} = \tan 18^\circ \Rightarrow R = \frac{14,4}{\tan 18^\circ} - 19,2 = 25,12 \Omega$$

$$\dot{Z} = 44,32 + j14,4 = 46,6 \angle 18^\circ \Omega$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{100 \angle 0^\circ}{46,6 \angle 18^\circ} = 2,146 \angle -18^\circ \text{ A}$$

$$\dot{U}_p = \dot{I} \cdot \dot{Z}_p = 2,146 \angle -18^\circ \cdot 24 \angle 36,87^\circ = 51,5 \angle 18,87^\circ \text{ V}$$

$$\dot{I}_x = \dot{U}_p \cdot \dot{Y}_2 = 51,5 \angle 18,87^\circ \cdot 0,025 \angle -90^\circ = 1,287 \angle -71,3^\circ \text{ A}$$

$$\dot{U}_L = \dot{I}_x \cdot j\dot{X}_L = 1,287 \angle -71,3^\circ \cdot 90 \angle 90^\circ = 115,9 \angle 18,87^\circ \text{ V}$$

$$\dot{U}_C = \dot{I}_x \cdot (-j\dot{X}_C) = 1,287 \angle -71,3^\circ \cdot 50 \angle -90^\circ = 64,4 \angle -161,1^\circ \text{ V}$$

6.21. Na transformator nominalne snage $S_n = 22 \text{ kVA}$ priključeno je trošilo prividne snage $S = 15 \text{ kVA}$ i faktora snage $\cos \varphi = 0,7$.

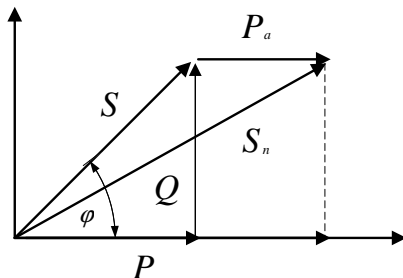
a) Kolika se maksimalna snaga čistog omskog trošila smije priključiti?

b) Kolika se snaga čistog omskog trošila smije priključiti da faktor snage iznosi $\cos \varphi' = 0,8$?

$$S_n = 22 \text{ kVA}$$

$$S = 15 \text{ kVA} \Rightarrow \dot{S} = 15 \angle 45,57^\circ = 10,5 + j10,712 \text{ kVA} = P + jQ$$

$$\cos \varphi = 0,7$$

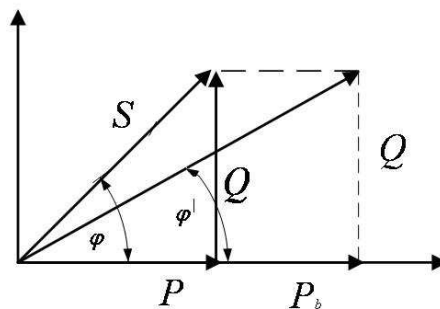


a)

$$S_n^2 = (P + P_a)^2 + Q^2$$

$$P_a = \sqrt{S_n^2 - Q^2} - P$$

$$= \sqrt{22^2 - 10,712^2} - 10,5 = 8,716 \text{ kW}$$



b)

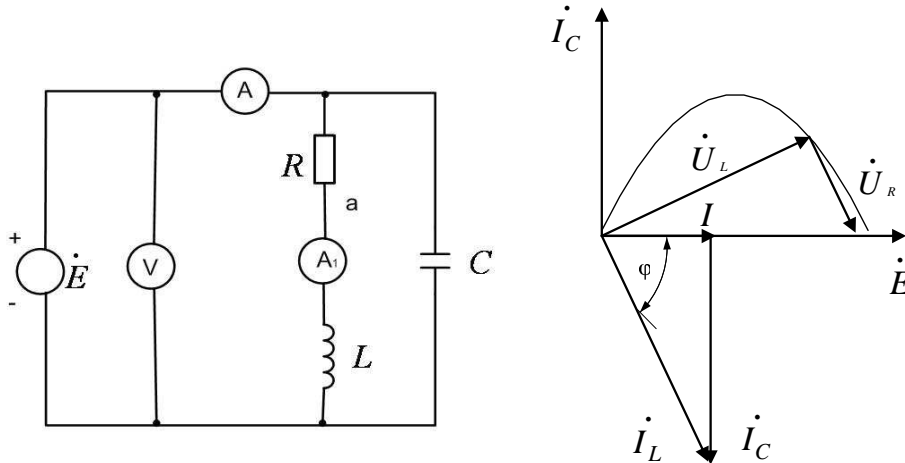
$$\cos \varphi' = 0,8 \Rightarrow \varphi' = 36,87^\circ$$

$$\tan \varphi' = \frac{Q}{P + P_b}$$

$$P_b = \frac{Q}{\tan \varphi'} - P = \frac{10,712}{\tan 36,87^\circ} - 10,5 = 3,783 \text{ kW}$$

7. Rezonancija

7.1. U mreži, prema slici nastaje rezonancija pri frekvenciji $\omega = 5000 \text{ rad/s}$, pri čemu su instrumenti izmjerili sljedeće vrijednosti: $U_V = 30 \text{ V}$, $I_A = 225 \text{ mA}$, $I_{A1} = 625 \text{ mA}$. Treba odrediti RLC i nacrtati topografski dijagram.



$$I_C = \sqrt{I_L^2 - I^2} = \sqrt{0,625^2 - 0,225^2} = 0,5831 \text{ A}$$

$$X_C = \frac{E}{I_C} = \frac{30}{0,5831} = 51,45 \Omega = \frac{1}{\omega C}$$

$$C = \frac{1}{5 \cdot 10^3 \cdot 51,45} = 3,887 \mu\text{F}$$

$$\tan \varphi = \frac{I_C}{I} = \frac{0,5831}{0,225} = 2,5915$$

$$\varphi = 68,9^\circ$$

$$Z_L = \frac{E}{I_L} = \frac{30}{0,625} = 48 \Omega \Rightarrow \dot{Z}_L = 48 \angle 68,9^\circ = 17,28 + j44,78 = R + jX_L$$

$$X_L = 44,78 = \omega L \Rightarrow L = \frac{X_L}{\omega} = \frac{44,78}{5 \cdot 10^3} = 8,96 \text{ mH}$$

7.2. U shemi prema slici struja i napon izvora su u rezonanciji pri $f = 60 \text{ Hz}$, pri čemu vatmetar pokazuje 100 W . Zadano je: $R = 16 \Omega$ i $X_L = 12 \Omega$. Treba odrediti kapacitet C te struje u vremenskom obliku, kao i njihove efektivne vrijednosti.

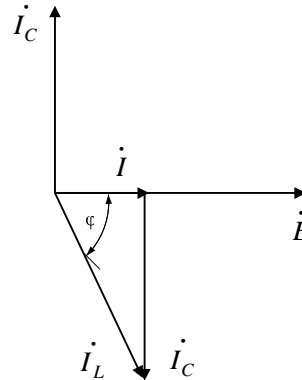
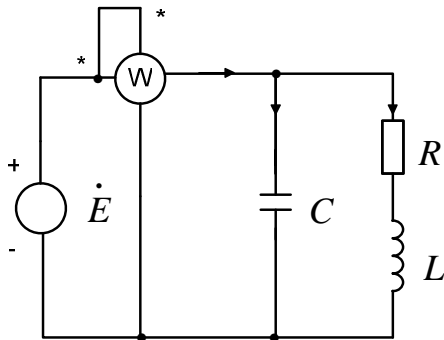
$$\omega = 2\pi f = 120\pi = 377 \text{ rad/s}$$

$$\dot{Z}_2 = R + jX_L = 16 + j12 = 20 \angle 36,87^\circ \Omega$$

$$\dot{Y}_2 = 0,05 \angle -36,87^\circ = 0,04 - j0,03 \text{ S}$$

$$\dot{Y} = \dot{Y}_1 + \dot{Y}_2 = j\omega C = +j0,04 - j0,03$$

$$I_m = \left\{ \dot{Y} \right\} = 0 \Rightarrow \omega C = 0,03 \Rightarrow C = \frac{0,03}{377} = 79,58 \mu F$$

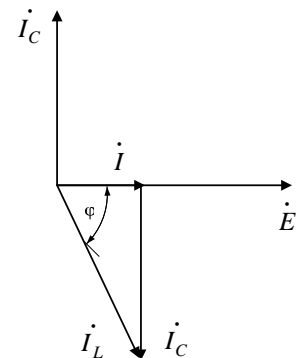
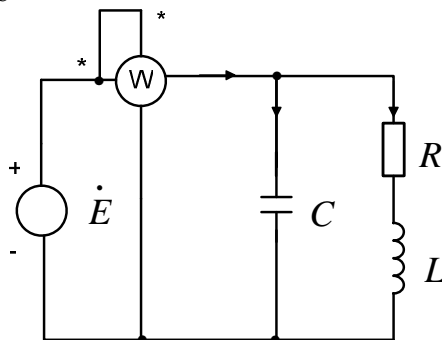


$$P = I_L^2 \cdot R \Rightarrow I_L = \sqrt{\frac{P}{R}} = \sqrt{\frac{100}{16}} = 2,5 \text{ A}$$

$$\cos \varphi = \frac{I}{I_L} \Rightarrow I = I_L \cdot \cos \varphi = 2,5 \cdot \cos 36,87^\circ = 2 \text{ A}$$

$$\sin \varphi = \frac{I_C}{I_L} \Rightarrow I_C = I_L \cdot \sin \varphi = 2,5 \cdot \sin 36,87^\circ = 1,5 \text{ A}$$

7.3. U shemi na slici u rezonanciji poznata je snaga $P = 80 \text{ W}$ i struje $I = 4 \text{ A}$ i $I_L = 5 \text{ A}$; treba odrediti R , X_L i X_C .



$$P = I_L^2 \cdot R \Rightarrow R = \frac{P}{I_L^2} = \frac{80}{5^2} = 3,2 \Omega$$

$$I_C = \sqrt{I_L^2 - I^2} = \sqrt{25 - 16} = 3 \text{ A}$$

$$\cos \varphi = \frac{I}{I_L} = \frac{4}{5} = 0,8 \Rightarrow \varphi = 36,87^\circ$$

$$\tan \varphi = \frac{X_L}{R}$$

$$X_L = R \cdot \tan \varphi = 3,2 \cdot \tan 36,87^\circ = 2,4 \Omega$$

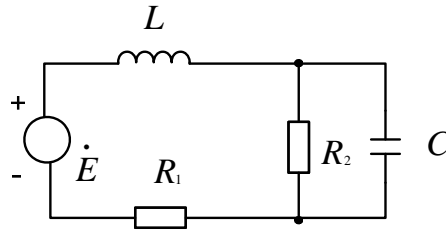
$$\dot{Z}_L = R + jX_L = 3,2 + j2,4 = 4 \angle 36,87^\circ \Omega$$

$$\dot{E} = \dot{I}_L \cdot \dot{Z}_L = 5 \angle -36,87^\circ \cdot 4 \angle 36,87^\circ$$

$$= 20 \angle 0^\circ = \dot{I}_C (-jX_C) = 3 \angle 90^\circ \cdot X_C \angle -90^\circ$$

$$3X_C = 20 \Rightarrow X_C = \frac{20}{3} = 6,6 \Omega$$

7.4. U shemi na slici nastaje rezonancija pri frekvenciji $f = 400 \text{ Hz}$. Ako je zadano $E = 50 \text{ V}$, $R_1 = R_2 = 20 \Omega$ i $C = 10 \mu\text{F}$, treba odrediti L i struju i_C .



$$\omega = 2\pi f = 800\pi = 2513,27 \text{ rad/s}$$

$$\omega C = 2513,27 \cdot 10 \cdot 10^{-6} = 0,0251327 \text{ S}$$

$$\dot{Y}_p = \frac{1}{R_2} + j\omega C = 0,05 + j0,0251327 = 0,05596 \angle 26,687^\circ \text{ S}$$

$$\dot{Z}_p = \frac{1}{\dot{Y}_p} = \frac{1}{0,05596 \angle 26,687^\circ} = 17,869 \angle -26,687^\circ \Omega$$

$$\dot{Z}_p = 15,966 - j8,025 \Omega$$

$$\dot{Z} = R_1 + \dot{Z}_p + jX_L = 20 + 15,966 - j8,025 + jX_L = 35,966 \Omega$$

$$I_m = \left\{ \dot{Z} \right\} = 0 \Rightarrow X_L = 8,025 \Omega = \omega L$$

$$L = \frac{8,025}{2513,27} = 3,1932 \text{ mH}$$

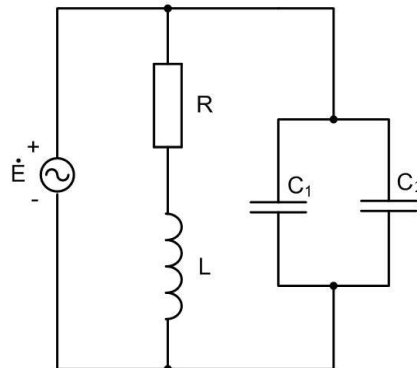
$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{50 \angle 0^\circ}{35,966} = 1,39 \text{ A}$$

$$\dot{U}_p = \dot{I} \cdot \dot{Z}_p = 1,39 \angle 0^\circ \cdot 17,869 \angle -26,687^\circ = 24,84 \angle -26,687^\circ \text{ V}$$

$$\dot{I}_C = \frac{\dot{U}_p}{-jX_C} = 24,84 \angle -26,687^\circ \cdot 0,02513 \angle 90^\circ = 0,6244 \angle 63,31^\circ \text{ A}$$

$$i_C = 0,883 \cdot \sin(2513,27t + 63,31^\circ) \text{ A}$$

7.5. U shemi na slici zadano je $e = \sin 10^5 t$ V, $R = 10 \Omega$, $L = 0.4$ mH i $C_1 = 0.1 \mu\text{F}$. Treba odrediti C_2 tako da mreža bude u rezonanciji. Kolika je pri tome radna snaga?



$$\dot{E} = \frac{1}{\sqrt{2}} \underline{0^\circ} = 0,7071 \text{ V}$$

$$X_L = \omega L = 10^5 \cdot 0,4 \cdot 10^{-3} = 40 \Omega$$

$$X_{C_1} = \frac{1}{\omega C_1} = \frac{1}{10^5 \cdot 0,1 \cdot 10^{-6}} = 100 \Omega$$

$$\dot{Z}_1 = R + jX_L = 10 + j40 = 41,231 \underline{75,9^\circ} \Omega$$

$$\dot{Y}_1 = \frac{1}{\dot{Z}_1} = 0,024253 \underline{-75,96^\circ} = 5,882 \cdot 10^{-3} - j0,02353 \text{ S}$$

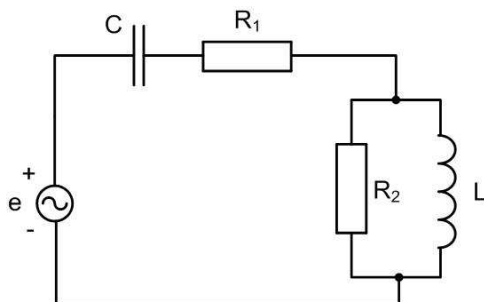
$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 = 5,882 \cdot 10^{-3} - j0,02353 + j\omega C \text{ S}$$

$$I_m \left\{ \dot{Y}_p \right\} = 0 \rightarrow \omega C = 0,02353 \text{ S}$$

$$C = \frac{0,02353}{10^5} = 235,29 \text{ nF} = C_1 + C_2 \rightarrow C_2 = 0,1353 \mu\text{F}$$

$$P = E^2 Y_p = 0,7071^2 \cdot 5,882 \cdot 10^{-3} = 2,94 \text{ mW}$$

7.6. U shemi na slici zadano je $e = 21 \sqrt{2} \sin 10^3 t$ V, $R_1 = 10 \Omega$, $R_2 = 40 \Omega$, $L = 100$ mH. Treba odrediti C takav da mreža bude u rezonanciji. Koliki je tada napon U_L ?



$$\dot{E} = 21 \underline{0^\circ} \text{ V}$$

$$\omega = 10^3 \text{ rad/s}$$

$$X_L = \omega L = 10^3 \cdot 0,1 = 100 \Omega$$

$$\dot{Y}_p = \frac{1}{R_2} - j \frac{1}{X_L} = \frac{1}{40} - j \frac{1}{100} = 0,025 - j0,01 \text{ S} = 0,026926 \angle -21,8^\circ \text{ S}$$

$$\dot{Z}_p = 37,139 \angle 21,8^\circ = 34,483 + j13,793 \Omega$$

$$\dot{Z} = R_1 + \dot{Z}_p - jX_C = 10 + 34,483 + j13,793 - jX_C = 44,483 \Omega$$

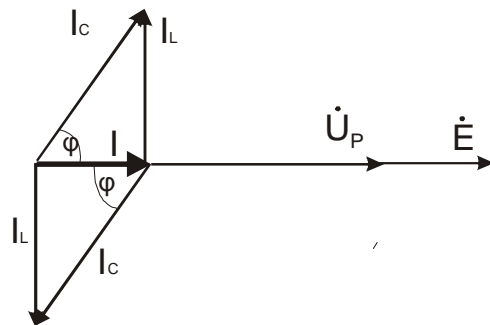
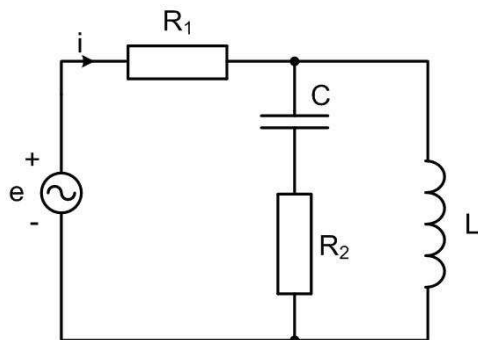
$$I_m \left\{ \dot{Z} \right\} = 0 \rightarrow X_C = 13,793 = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega X_C} = \frac{1}{10^3 \cdot 13,793} = 72,5 \mu\text{F}$$

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{21 \angle 0^\circ}{44,483 \angle 0^\circ} = 0,4721 \text{ A}$$

$$\dot{U}_L = \dot{I} \cdot \dot{Z}_p = 0,4721 \cdot 37,139 \angle 21,8^\circ = 17,533 \angle 21,8^\circ \text{ V}$$

7.7. U shemi na slici, koja je u rezonanciji pri frekvenciji $f = 400 \text{ Hz}$, poznato je da je snaga na otporu R_1 tri puta manja od snage na otporu R_2 , dok je snaga izvora $P = 1024 \text{ W}$. Zadano je $R_1 = R_2 = 100 \Omega$. Treba odrediti L , C i EMS e .



$$\omega = 2\pi f = 2\pi 400 = 2513,27 \text{ rad/s}$$

$$P = 1024 \text{ W}$$

$$P_1 = \frac{P_2}{3}$$

$$P_1 + P_2 = P$$

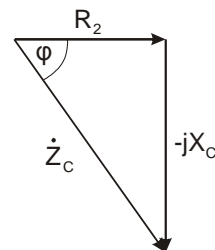
$$\frac{P_2}{3} + P_2 = P \quad / \cdot 3$$

$$P_2 + 3P_2 = 3P$$

$$4P_2 = 3P$$

$$P_2 = \frac{3}{4}P = 0,75 \cdot 1024 = 768 \text{ W} ; P_1 = \frac{P_2}{3} = 256 \text{ W}$$

$$I = \sqrt{\frac{P_1}{R_1}} = \sqrt{\frac{256}{100}} = 1,6 \text{ A}$$



$$I_C = \sqrt{\frac{P_2}{R_2}} = \sqrt{\frac{768}{100}} = 2,7713 \text{ A}$$

$$I_L = \sqrt{I_C^2 - I^2} = \sqrt{2,7713^2 - 1,6^2} = 2,26274 \text{ A}$$

$$\operatorname{tg}|\varphi| = \frac{I_L}{I} = \frac{2,26274}{1,6} = 1,4142 \rightarrow \varphi = 54,736^\circ$$

$$\operatorname{tg}|\varphi| = \frac{X_C}{R_2} \rightarrow X_C = R_2 \operatorname{tg}|\varphi| = 100 \cdot 1,4142 = 141,42 \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{2513,27 \cdot 141,42} = 2,8135 \mu\text{F}$$

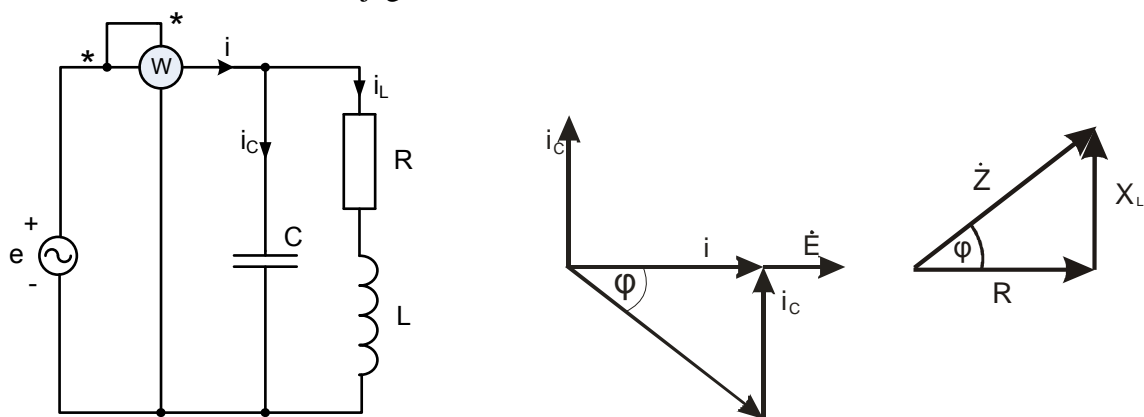
$$\dot{Z}_C = R_2 - jX_C = 100 - j141,42 = 173,2 \angle -54,736^\circ \Omega$$

$$\dot{U}_p = \dot{I}_C \cdot \dot{Z}_C = 2,7713 \angle 54,736^\circ \cdot 173,2 \angle -54,736^\circ = 480 \angle 0^\circ \text{ V}$$

$$jX_L = \frac{\dot{U}_p}{\dot{I}_L} = \frac{480 \angle 0^\circ}{2,26274 \angle -90^\circ} = 212,132 \angle 90^\circ \rightarrow L = \frac{X_L}{\omega} = \frac{212,132}{2513,27} = 84,4 \text{ mH}$$

$$\dot{E} = \dot{I} R_1 + \dot{U}_p = 1,6 \cdot 100 + 480 = 640 \text{ V} \rightarrow e = 905,1 \sin(2513,7t) \text{ V}$$

7.8. U shemi na slici poznato je $e = 64 \sin 2000t \text{ V}$, $i = 5 \sin 2000t \text{ A}$ i $R = 8 \Omega$. Treba odrediti L i C te nacrtati fazorski dijagram.



$$P = I \cdot E = \frac{64}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = 160 \text{ W} = I_L^2 \cdot R \rightarrow I_L = \sqrt{\frac{P}{R}} = \sqrt{\frac{160}{8}} = 4,4721 \text{ A}$$

$$I_C = \sqrt{I_L^2 - I^2} = \sqrt{4,4721^2 - \frac{25}{2}} = 2,7386 \text{ A}$$

$$\cos \varphi = \frac{I}{I_L} = \frac{5}{\sqrt{2} \cdot 4,4721} = 0,79057 \rightarrow \varphi = 37,761^\circ$$

$$X_L = R \cdot \operatorname{tg} \varphi = 8 \cdot \operatorname{tg} 37,761^\circ = 6,19677 = \omega L$$

$$L = \frac{X_L}{\omega} = \frac{6,19677}{2000} = 3,098 \text{ mH}$$

$$\dot{Z}_L = R + jX_L = 10,119 \angle 37,761^\circ \Omega$$

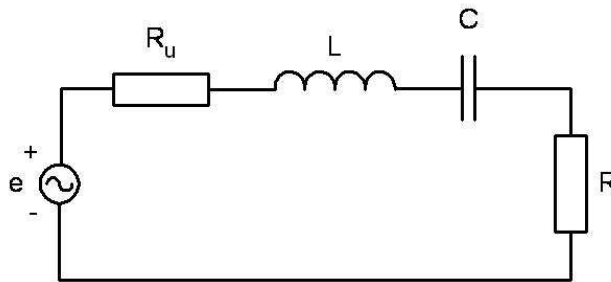
$$\dot{U}_p = \dot{I}_L \cdot \dot{Z}_L = 4,4721 \angle -37,761^\circ \cdot 10,119 \angle 37,731^\circ = 45,2548 \angle 0^\circ \text{ V}$$

$$= \dot{I}_C \cdot (-jX_C) = 2,7386 \angle 90^\circ \cdot X_C \angle -90^\circ$$

$$X_C = \frac{45,2548}{2,7386} = 16,525 \Omega$$

$$C = \frac{1}{\omega X_C} = \frac{1}{2000 \cdot 16,525} = 30,26 \mu\text{F}$$

7.9. U shemi na slici treba odrediti frekvenciju naponskog izvora pri kojoj je snaga na otporu R maksimalna. Zadano: $E_m = 100 \text{ V}$, $R_u = 10 \Omega$, $L = 2 \text{ mH}$, $C = 0,88 \text{ F}$ i $R = 40 \Omega$.



$$e = E_m \sin \omega t$$

$$P = I^2 R$$

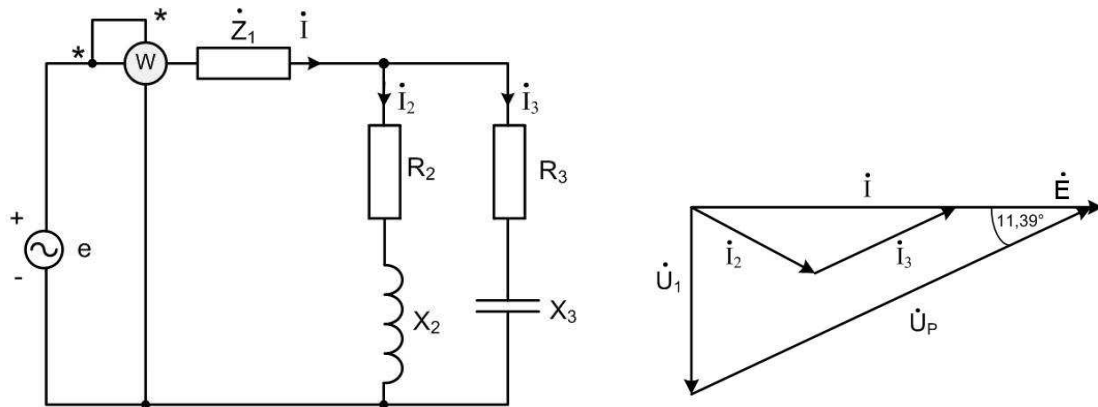
$$I = \frac{E}{\sqrt{(R_u + R)^2 + (X_L - X_C)^2}}$$

$$\left. \begin{array}{l} P \uparrow \Rightarrow I \uparrow \Rightarrow Z \downarrow \Rightarrow X_L - X_C = 0 \\ \omega L = \frac{1}{\omega C} \end{array} \right\}$$

$$\omega^2 = \frac{1}{LC} \rightarrow \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \cdot 10^{-3} \cdot 0,88}} = 23,836 = 2\pi f$$

$$f = \frac{23,836}{2\pi} = 3,7937 \text{ Hz}$$

7.10. Treba odrediti impedanciju \dot{Z}_1 u shemi na slici tako da vatmetar pokazuje maksimalnu snagu. Zadano: $R_2 = 8 \Omega$, $X_2 = 6 \Omega$, $R_3 = 12 \Omega$, $X_3 = 5 \Omega$ i $E = 110 \text{ V}$. Treba nacrtati fazorski dijagram.



$$\dot{Y}_2 = \frac{1}{R_2 + jX_2} = \frac{1}{8 + j6} = 0,1 \angle -36,87^\circ = 0,08 - j0,06 \text{ S}$$

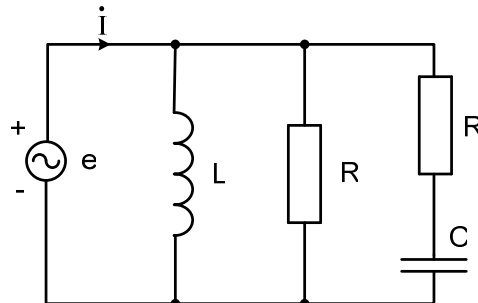
$$\dot{Y}_3 = \frac{1}{R_3 - jX_3} = \frac{1}{12 - j5} = 0,077 \angle 22,6^\circ = 0,071 + j0,0296 \text{ S}$$

$$\dot{Y}_p = \dot{Y}_2 + \dot{Y}_3 = 0,151 - j0,0304 = 0,154 \angle -11,39^\circ$$

$$\dot{Z}_p = \frac{1}{\dot{Y}_p} = 6,492 \angle 11,387^\circ = 6,364 + j1,2818 \Omega$$

$$\dot{Z} = \dot{Z}_1 + \dot{Z}_p \rightarrow \dot{Z}_1 = -j1,2818 \Omega$$

7.11. U mreži na slici treba odrediti C tako da napon i struja budu u fazi.
Zadano: $R = 10 \Omega$, $L = 20 \text{ mH}$, $f = 223 \text{ Hz}$.



$$\omega = 2\pi f = 2\pi \cdot 223 = 1401,15 \text{ rad/s}$$

$$X_L = \omega L = 1401,15 \cdot 20 \cdot 10^{-3} = 28,023 \Omega$$

$$\dot{Y} = -j \frac{1}{X_L} = 0,03568 \angle -90^\circ \text{ S}$$

$$\dot{Y}_2 = \frac{1}{R - jX_C} = \frac{1}{10 - jX_C} \cdot \frac{10 + jX_C}{10 + jX_C} = \frac{10 + jX_C}{10^2 + X_C^2}$$

$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 \quad ; \quad I_m \left\{ \dot{Y}_p \right\} = 0$$

$$\frac{X_C}{10^2 + X_C^2} = \frac{1}{X_L} \quad / \cdot X_L (10^2 + X_C^2)$$

$$X_C X_L = 10^2 + X_C^2$$

$$X_C^2 - 28,023 X_C + 100 = 0$$

$$X_{C_1} = 23,826 \, \Omega \quad \rightarrow \quad C_1 = \frac{1}{\omega X_{C_1}} = \frac{1}{1401,15 \cdot 23,826} = 29,95 \, \mu\text{F}$$

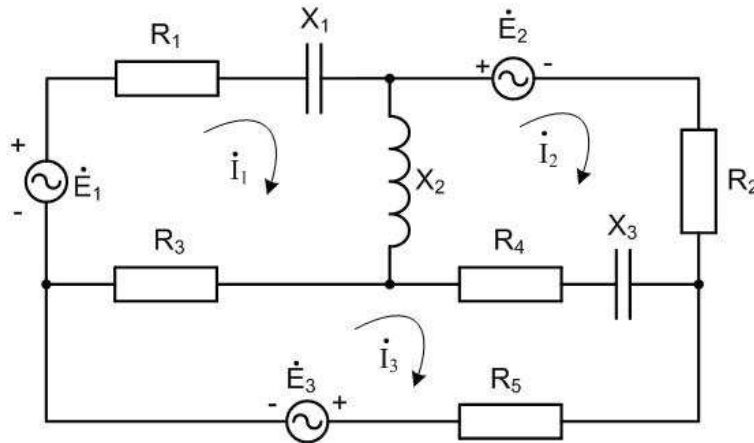
$$X_{C_2} = 4,1971 \, \Omega \quad \rightarrow \quad C_2 = \frac{1}{\omega X_{C_2}} = \frac{1}{1401,15 \cdot 4,1971} = 0,17 \, \text{mF}$$

8. Metode i teoremi linearnih električnih mreža

8.1. Treba odrediti radnu jalovu snagu u shemi na slici.

Zadano: $R_1 = 2 \Omega$, $R_2 = 10 \Omega$, $R_3 = 5 \Omega$, $R_4 = 2 \Omega$, $R_5 = 10 \Omega$, $X_1 = 2 \Omega$, $X_2 = 5 \Omega$, $X_3 = 2 \Omega$,

$\dot{E}_1 = 10 \angle 0^\circ \text{ V}$, $\dot{E}_2 = 5 \angle 30^\circ \text{ V}$, $\dot{E}_3 = 10 \angle 90^\circ \text{ V}$



$$(1) \quad \dot{I}_1(7 + j3) - \dot{I}_2 j5 - \dot{I}_3 5 = 10$$

$$(2) \quad -j5\dot{I}_1 + \dot{I}_2(12 + j3) - \dot{I}_3(2 - j2) = 5 \angle -150^\circ$$

$$(3) \quad -5\dot{I}_1 - \dot{I}_2(2 - j2) + \dot{I}_3(17 - j2) = 10 \angle -90^\circ \rightarrow \dot{I}_1 = -\dot{I}_2(0,4 - j0,4) + \dot{I}_3(3,4 - j0,4) + j2$$

$$(1) + (3) \rightarrow (7 + j3) \left[-0,4\dot{I}_2 + j0,4\dot{I}_2 + 3,4\dot{I}_3 - j0,4\dot{I}_3 + j2 \right] - \dot{I}_2 j5 - \dot{I}_3 5 = 10$$

$$\dot{I}_3 = \frac{16 - j14}{20 + j7,4} + \frac{4 + j3,4}{20 + j7,4} \dot{I}_2 = 0,99696 \angle -61,49^\circ + 0,246177 \angle 20,06^\circ \dot{I}_2$$

$$(3) \rightarrow \dot{I}_1 = -\dot{I}_2(0,4 - j0,4) + \left[0,99696 \angle -61,49^\circ + 0,246177 \angle 20,06^\circ \dot{I}_2 \right] \cdot 3,42345 \angle -6,71^\circ + j2$$

$$= 0,72797 \angle 54,764^\circ \dot{I}_2 + 1,72425 \angle -42,684^\circ$$

$$(2) \quad 5 \angle -90^\circ \left[0,72797 \angle 54,764^\circ \dot{I}_2 + 1,72425 \angle -42,684^\circ \right] + \dot{I}_2(12 + j3) +$$

$$+ 2,82843 \angle 135^\circ \left[0,99696 \angle -61,49^\circ + 0,246177 \angle 20,06^\circ \dot{I}_2 \right] = 5 \angle -150^\circ$$

$$\dot{I}_2 = 0,0931 \angle 53,02^\circ \text{ A} \rightarrow \dot{S}_2 = \dot{U}_2 \dot{I}_2^* = 5 \angle -150^\circ \cdot 0,0931 \angle -53,02^\circ = -0,4284 + j0,18203 \text{ VA}$$

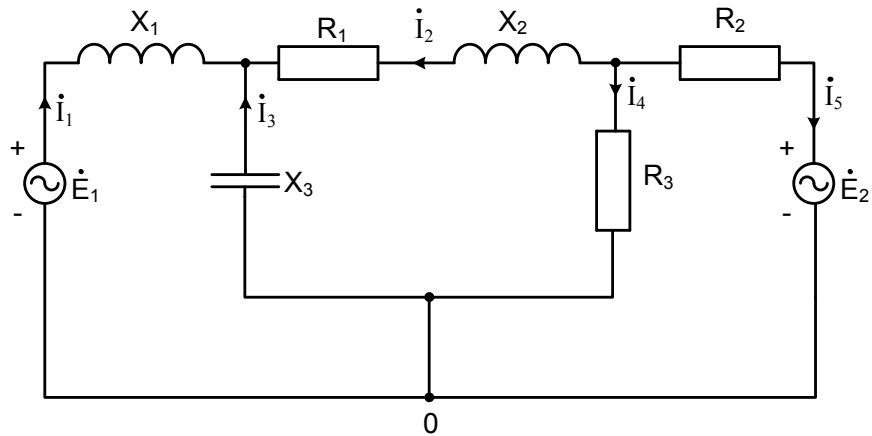
$$\dot{I}_1 = 1,6656 \angle -41,53^\circ \text{ A} \rightarrow \dot{S}_1 = \dot{U}_1 \dot{I}_1^* = 10 \cdot 1,6656 \angle 41,53^\circ = 12,4688 + j11,043 \text{ VA}$$

$$\dot{I}_3 = 0,981 \angle -60,54^\circ \text{ A} \rightarrow \dot{S}_3 = \dot{U}_3 \dot{I}_3^* = 10 \angle -90^\circ \cdot 0,981 \angle 60,54^\circ = 8,5416 - j4,825 \text{ VA}$$

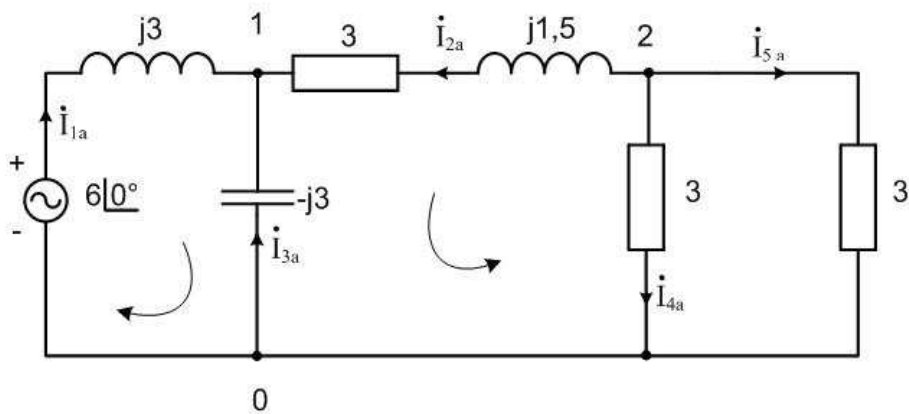
$$\dot{S} = 20,6 + j6,4 \text{ VA}$$

8.2. U shemi na slici treba odrediti sve struje grana i radnu snagu mreže.

Zadano: $R_1 = R_2 = R_3 = 3 \Omega$, $X_1 = X_3 = 3 \Omega$, $X_2 = 1,5 \Omega$, $\dot{E}_1 = 6 \angle 0^\circ \text{ V}$, $\dot{E}_2 = 6 \angle 90^\circ \text{ V}$.



a)

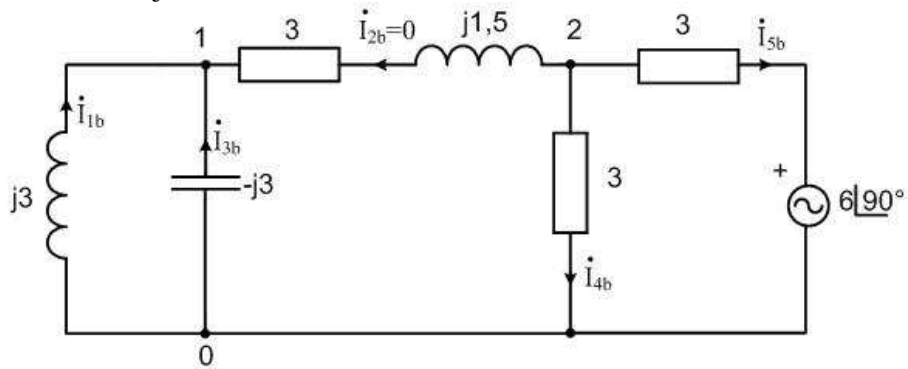


$$1) \quad \dot{I}_{1a}(j3 - j3) + \dot{I}_{2a}(-j3) = 6 \angle 0^\circ \rightarrow \dot{I}_{2a} = \frac{6}{3 \angle -90^\circ} = j2 \text{ A}$$

$$\begin{aligned} 2) \quad & -j3\dot{I}_{1a} + \dot{I}_{2a}(3 - j3 + j1,5 + 1,5) = 0 \\ & -j3\dot{I}_{1a} + j2(4,5 - j1,5) = 0 \\ & -j3\dot{I}_{1a} + 3 + j9 = 0 \quad / : 3 \\ & -j\dot{I}_{1a} = -1 - j3 \quad / \cdot j \\ & \dot{I}_{1a} = 3 - j \\ & \dot{I}_{3a} = -\dot{I}_{1a} - \dot{I}_{2a} = -3 + j - j2 = -3 - j \text{ A} \\ & \dot{I}_{5a} = \dot{I}_{4a} = -j \text{ A} \end{aligned}$$

b)

rezonancija



$$\dot{I}_{5b} = \frac{-j6}{6} = -j = -\dot{I}_{4b}$$

$$\dot{\varphi}_{1b} = j3 \rightarrow \dot{I}_{1b} = -\frac{\dot{\varphi}_{1b}}{j3} = \frac{-j3}{j3} = -1 \text{ A}$$

$$\dot{I}_{3b} = \frac{-\dot{\varphi}_{1b}}{-j3} = \frac{-j3}{-j3} = 1 \text{ A}$$

superponiranjem a) + b) dobivamo:

$$\dot{I}_1 = \dot{I}_{1a} + \dot{I}_{1b} = 3 - j - 1 = 2 - j = 2,24 \angle -26,56^\circ \text{ A}$$

$$\dot{I}_2 = \dot{I}_{2a} = j2$$

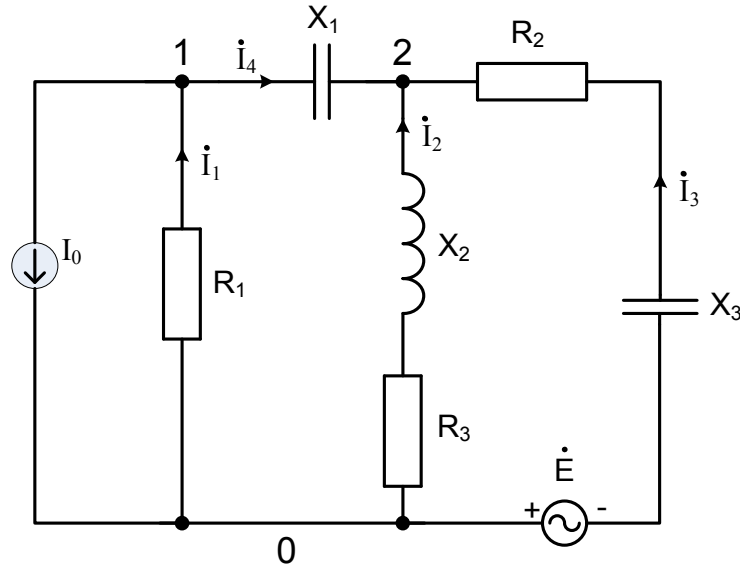
$$\dot{I}_3 = \dot{I}_{3a} + \dot{I}_{3b} = -3 - j + 1 = -2 - j = 2,24 \angle 206,56^\circ \text{ A}$$

$$\dot{I}_5 = \dot{I}_{5a} + \dot{I}_{5b} = -j - j = -2j \text{ A}$$

$$P = (I_2^2 + I_4^2 + I_5^2) \cdot 3 = (4 + 0 + 4) \cdot 3 = 24 \text{ W}$$

8.3. U shemi na slici treba odrediti struje svih grana.

Zadano: $R_1 = R_2 = 10 \, \Omega$, $R_3 = 5 \, \Omega$, $X_1 = X_3 = 10 \, \Omega$, $X_2 = 5 \, \Omega$, $\dot{I}_0 = 1 \angle 0^\circ \text{ A}$, $\dot{E} = 10 \angle 0^\circ \text{ V}$.



$$(1) \quad \dot{\varphi}_1 \left(\frac{1}{10} + \frac{j}{10} \right) - \dot{\varphi}_2 \frac{j}{10} = -1$$

$$(2) \quad -\dot{\varphi}_1 \frac{j}{10} + \dot{\varphi}_2 \left(\frac{j}{10} + \frac{1}{5 + j5} + \frac{1}{10 - j10} \right) = \frac{-10}{10 - j10}$$

$$(1) \rightarrow \dot{\varphi}_2 \cdot 0,1 \angle 90^\circ = \dot{\varphi}_1 \cdot 0,1 \sqrt{2} \angle 45^\circ + 1 \quad / \cdot 10 \angle -90^\circ$$

$$\dot{\varphi}_2 = \dot{\varphi}_1 \cdot \sqrt{2} \angle -45^\circ + 10 \angle -90^\circ$$

$$(2) \rightarrow -\dot{\varphi}_1 \cdot 0,5j + \left(\dot{\varphi}_1 \cdot \sqrt{2} \angle -45^\circ + 10 \angle -90^\circ \right) \cdot 0,15811 \angle 18,4349^\circ = -0,5 - j0,5$$

$$\frac{1}{5\sqrt{2} \angle 45^\circ} = \frac{\sqrt{2} \angle -45^\circ}{10} = 0,1 - j0,1$$

$$\frac{1}{10\sqrt{2} \angle -45^\circ} = \frac{\sqrt{2} \angle 45^\circ}{20} = 0,05 + j0,05$$

$$j0,1 + 0,1 - j0,1 + 0,05 + j0,05 = 0,15 + j0,05 = 0,15811 \angle 18,4349^\circ$$

$$\frac{-1}{\sqrt{2} \angle 45^\circ} = \frac{-\sqrt{2} \angle 45^\circ}{2} = 0,5\sqrt{2} \angle -135^\circ = -0,5 - j0,5$$

$$\dot{\varphi}_1(-0,1j + 0,2 - j0,1) + 0,5 - j1,5 = -0,5 - j0,5$$

$$\dot{\varphi}_1 = \frac{-1+j}{0,2-j0,2} = \frac{\sqrt{2}|135^\circ}{0,2\sqrt{2}|-45^\circ} = 5|180^\circ = -5 \text{ V}$$

$$\dot{\varphi}_2 = \dot{\varphi}_1\sqrt{2}|-45^\circ - j10 = 5|180^\circ \cdot \sqrt{2}|-45^\circ - j10 = 5\sqrt{2}|135^\circ - j10 = -5 + j5 - j10 = -5 - j5 \text{ V}$$

$$\dot{I}_1 = \frac{-\dot{\varphi}_1}{10} = \frac{5}{10} = 0,5 \text{ A}$$

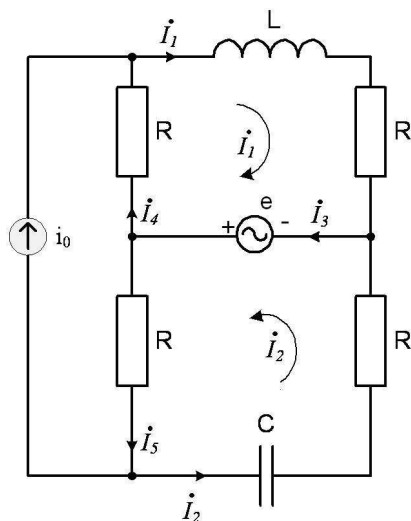
$$\dot{I}_4 = \frac{\dot{\varphi}_1 - \dot{\varphi}_2}{-j10} = \frac{-5 + 5 + j5}{10|-90^\circ} = \frac{5|90^\circ}{10|-90^\circ} = 0,5|180^\circ = -0,5 \text{ A}$$

$$\dot{I}_2 = \frac{-\dot{\varphi}_2}{5+j5} = \frac{5\sqrt{2}|45^\circ}{5\sqrt{2}|45^\circ} = 1 \text{ A}$$

$$\dot{I}_3 = \frac{-10 - \dot{\varphi}_2}{10 - j10} = \frac{-10 + 5 + j5}{10 - j10} = \frac{-5 + j5}{10\sqrt{2}|-45^\circ} = \frac{5\sqrt{2}|135^\circ}{10\sqrt{2}|-45^\circ} = 0,5|180^\circ = -5 \text{ A}$$

8.4. U mreži na slici treba odrediti struje svih grana i radnu snagu svih izvora.

Zadano: $e = -100\sqrt{2} \cos 10^4 t \text{ V}$, $i_0 = 10\sqrt{2} \sin 10^4 t \text{ A}$, $R = 10 \Omega$, $L = 4 \text{ mH}$, $C = 2,5 \mu\text{F}$.



$$e = -100\sqrt{2} \sin(\omega t + 90^\circ) = 100\sqrt{2} \cdot \sin(\omega t - 90^\circ)$$

$$\dot{I}_0 = 10 \text{ A}$$

$$\sin(\alpha + 90^\circ) = \cos \alpha$$

$$\dot{E} = 100|-90^\circ = -j100$$

$$\omega = 10^4 \text{ rad/s}$$

$$X_L = \omega L = 10^4 \cdot 4 \text{ mH} = 40 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{10^4 \cdot 2,5 \cdot 10^{-6}} = 40 \Omega$$

$$(1) \quad \dot{I}_1(2R + jX_L) - \dot{I}_0 R = -j100$$

$$\dot{I}_1 = \frac{10 \cdot 10 - j100}{20 + j40} = \frac{141,42|-45^\circ}{44,72|63,435^\circ} = 3,162|-108,43^\circ = -1 - j3 \text{ A}$$

$$(2) \quad \dot{I}_2(2R - jX_C) + \dot{I}_0 R = -j100$$

$$\dot{I}_2 = \frac{-100 - j100}{20 - j40} = \frac{141,42 \angle -135^\circ}{47,72 \angle -63,43^\circ} = 3,162 \angle -71,565^\circ = 1 - j3 \text{ A}$$

$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2 = -1 - j3 + 1 - j3 = -j6 \text{ A}$$

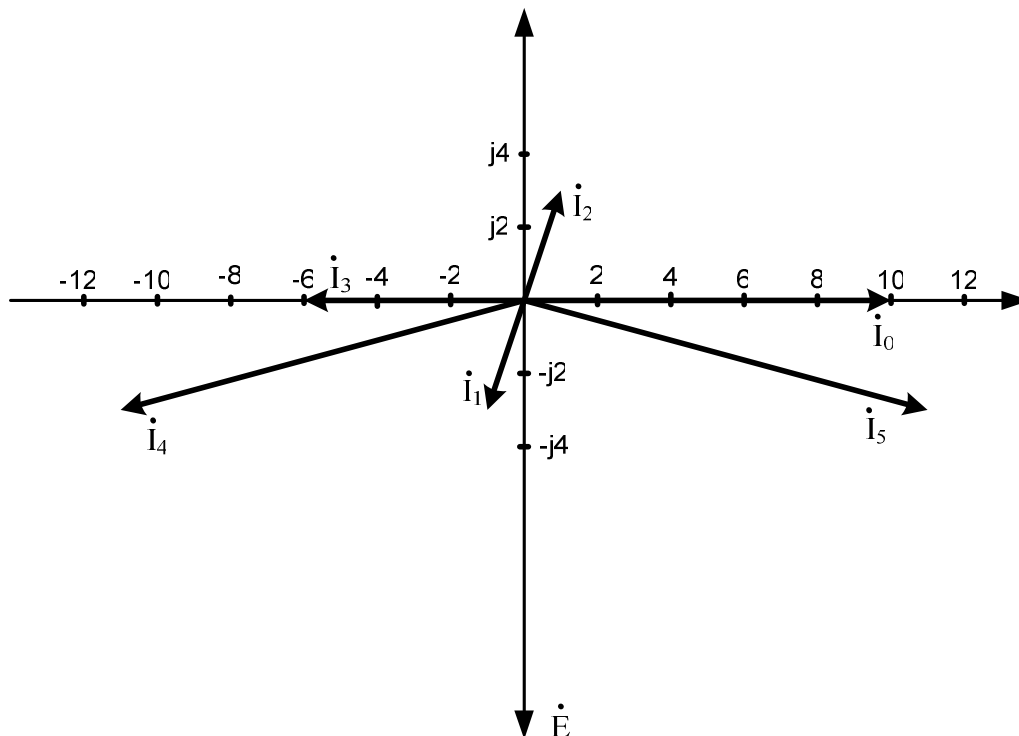
$$\dot{I}_4 = \dot{I}_1 - \dot{I}_0 = -1 - j3 - 10 = -11 - j3 = 11,4 \angle -164,74^\circ \text{ A}$$

$$\dot{I}_5 = \dot{I}_2 + \dot{I}_0 = 1 - j3 + 10 = 11 - j3 = 11,4 \angle -15,26^\circ \text{ A}$$

$$\dot{S}_e = \dot{E} \cdot \dot{I}_3^* = 100 \angle -90^\circ \cdot 6 \angle 90^\circ = 600 \text{ W}$$

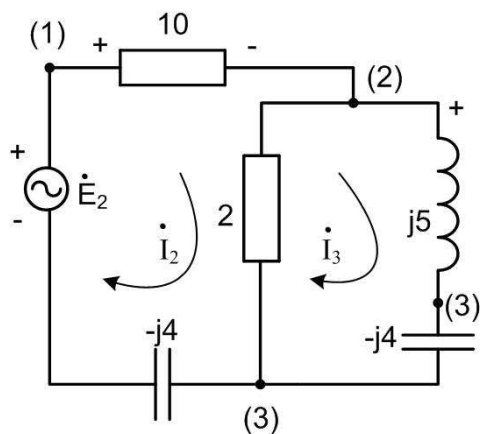
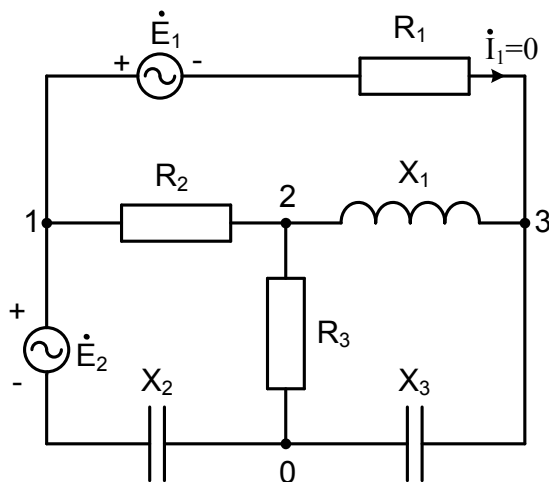
$$\dot{U}_0 = -\dot{I}_4 R + \dot{I}_5 R = (\dot{I}_5 - \dot{I}_4) R = (11 - j3 + 11 + j3) 10 = 220 \text{ V}$$

$$\dot{S}_{i_0} = \dot{U}_0 \cdot \dot{I}_0^* = 220 \angle 0^\circ \cdot 10 \angle 0^\circ = 2200 \text{ W}$$



8.5. U mreži na slici treba odrediti EMS \dot{E}_2 tako da je struja grane $\dot{I}_1 = 0$.

Zadano: $\dot{E}_1 = 50 \angle 0^\circ \text{ V}$; $R_1 = 5 \Omega$; $R_2 = 10 \Omega$; $R_3 = 2 \Omega$; $X_1 = 5 \Omega$; $X_2 = X_3 = 4 \Omega$



$$\left. \begin{aligned} (1) \quad -50 \angle 0^\circ &= -\dot{I}_2 \cdot 10 - \dot{I}_3 \cdot j5 \\ (2) \quad \dot{E}_2 &= \dot{I}_2(12 - j4) - \dot{I}_3 \cdot 2 \\ (3) \quad 0 &= -\dot{I}_2 \cdot 2 + \dot{I}_3(2 + j) \end{aligned} \right\} \text{konturne struje}$$

$$(1) \rightarrow \dot{I}_2 = 5 \angle 0^\circ - j0,5 \dot{I}_3 \rightarrow (3)$$

$$(1) i (3) \rightarrow 10 - j \dot{I}_3 = \dot{I}_3(2 + j)$$

$$10 = (2 + j2) \dot{I}_3$$

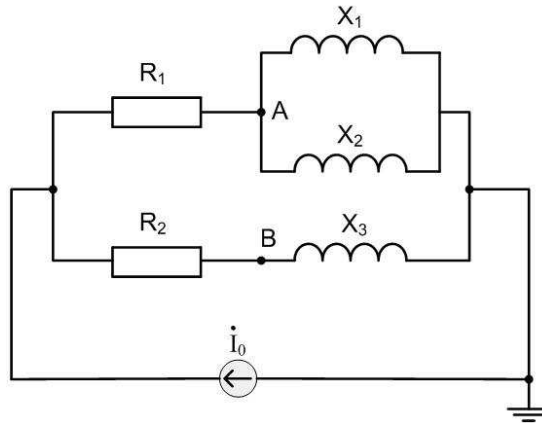
$$\dot{I}_3 = \frac{10}{2\sqrt{2} \angle 45^\circ} = \frac{5\sqrt{2}}{2} \angle -45^\circ = 2,5\sqrt{2} \angle -45^\circ = 2,5 - j2,5 \text{ A}$$

$$\begin{aligned} (1) \rightarrow \dot{I}_2 &= 5 + 2,5\sqrt{2} \angle -45^\circ \cdot 0,5 \angle -90^\circ = 5 + 1,25\sqrt{2} \angle -135^\circ = 5 - 1,25 - j1,25 = 3,75 - j1,25 = \\ &= 3,75 - j1,25 = 3,958 \angle -18,435^\circ \text{ A} \end{aligned}$$

$$\begin{aligned} (2) \rightarrow \dot{E}_2 &= 3,958 \angle -18,435^\circ \cdot 12,65 \angle -18,435^\circ - 2(2,5 - j2,5) = \\ &= 50 \angle -36,87^\circ - 5 + j5 = 40 - j30 - 5 + j5 = 35 - j25 = 43,01 \angle -35,54^\circ \text{ V} \end{aligned}$$

8.6. U mreži na slici treba odrediti napon \dot{U}_{AB} .

Zadano: $R_1 = R_2 = 2 \Omega$; $X_1 = 5 \Omega$, $X_2 = 10 \Omega$; $X_3 = 4 \Omega$; $\dot{I}_0 = 10 \angle 0^\circ \text{ A}$



$$\frac{j5 \cdot j10}{j5 + j10} = \frac{-50}{j15} = j\frac{10}{3} \Omega$$

$$\dot{Y}_1 = \frac{1}{2 + j\frac{10}{3}} = \frac{1}{3,8873 \angle 59,04^\circ} = 0,2572 \angle -59,04^\circ = 0,132353 - j0,22059 \text{ S}$$

$$\dot{Y}_2 = \frac{1}{2 + j4} = \frac{1}{4,472 \angle 63,435^\circ} = 0,22361 \angle -63,435^\circ \text{ S}$$

$$\dot{Y} = \dot{Y}_1 + \dot{Y}_2 = 0,232353 - j0,420588 = 0,4805 \angle -61,08^\circ \text{ S}$$

$$\dot{Z} = \frac{1}{\dot{Y}} = 2,081 \angle 61,08^\circ \Omega$$

$$\dot{U}_p = \dot{I}_0 \cdot \dot{Z} = 10 \angle 0^\circ \cdot 2,081 \angle 61,08^\circ = 20,81 \angle 61,08^\circ \text{ V}$$

$$\dot{I}_1 = \dot{U}_p \cdot \dot{Y}_1 = 20,81 \angle 61,08^\circ \cdot 0,2572 \angle -59,04^\circ = 5,35273 \angle 2,04^\circ = 5,35 + j0,191 \text{ A}$$

$$\dot{I}_2 = \dot{U}_p \cdot \dot{Y}_2 = 20,81 \angle 61,08^\circ \cdot 0,22361 \angle -63,435^\circ = 4,6533 \angle -2,353^\circ = 4,65 - j0,191 \text{ A}$$

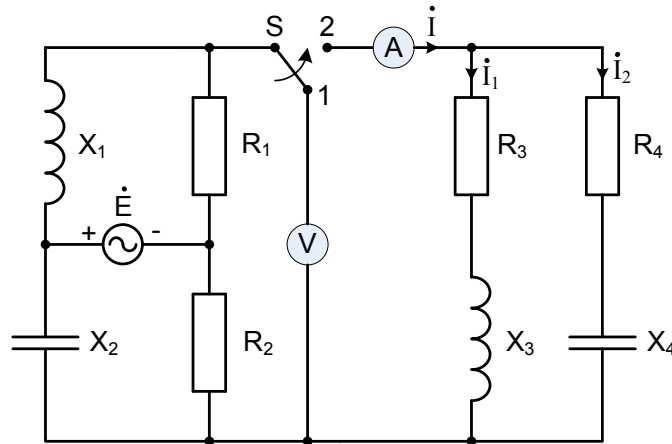
$$\dot{\varphi}_A = \dot{I}_1 \cdot j\frac{10}{3} = 5,3527 \angle 2,04^\circ \cdot \frac{10}{3} \angle 90^\circ = 17,842 \angle 92,04^\circ = -0,635 + j17,83 \text{ V}$$

$$\dot{\varphi}_B = \dot{I}_2 \cdot j4 = 4,6533 \angle -2,353^\circ \cdot 4 \angle 90^\circ = 18,61 \angle 87,65^\circ = 0,7643 + j18,598 \text{ V}$$

$$\dot{U}_{AB} = \dot{\varphi}_A - \dot{\varphi}_B = -1,4 - j0,77 = 1,6 \angle -151,3^\circ \text{ V}$$

8.7. U mreži na slici idealni voltmetar mjeri napon $U_V = 400$ V kad je sklopka u položaju 1. Koliku struju mjeri idealni ampermetar kada je preklopka u položaju 2? Kolike su efektivne vrijednosti struja I_1 , I_2 i EMS E ?

Zadano: $R_1 = R_2 = R_3 = R_4 = X_1 = X_2 = X_3 = X_4 = 10 \Omega$.



$$E_T = 400 \text{ V}$$

$$\dot{Y}_1 = \frac{1}{10} - \frac{j}{10} = 0,1 - j0,1 = 0,1\sqrt{2} \angle -45^\circ \text{ S}$$

$$\dot{Y}_2 = \frac{1}{10} + \frac{j}{10} = 0,1 + j0,1 = 0,1\sqrt{2} \angle 45^\circ \text{ S}$$

$$\dot{Z}_1 = \frac{10}{\sqrt{2}} \angle 45^\circ = 5\sqrt{2} \angle 45^\circ = 5 + j5 \Omega$$

$$\dot{Z}_2 = \frac{10}{\sqrt{2}} \angle -45^\circ = 5\sqrt{2} \angle -45^\circ = 5 - j5 \Omega$$

$$\dot{Z}_T = \dot{Z}_1 + \dot{Z}_2 = 5 + j5 + 5 - j5 = 10 \Omega$$

$$\dot{Y}_3 = \frac{1}{10 + j10} = \frac{1}{10\sqrt{2} \angle 45^\circ} = \frac{0,1\sqrt{2}}{2} \angle -45^\circ = 0,05\sqrt{2} \angle -45^\circ = 0,05 - j0,05 \text{ S}$$

$$\dot{Y}_4 = \frac{1}{10 - j10} = \frac{1}{10\sqrt{2} \angle -45^\circ} = \frac{0,1\sqrt{2}}{2} \angle 45^\circ = 0,05\sqrt{2} \angle 45^\circ = 0,05 + j0,05 \text{ S}$$

$$\dot{Y}_p = \dot{Y}_3 + \dot{Y}_4 = 0,05 - j0,05 + 0,05 + j0,05 = 0,1 \text{ S} \rightarrow \dot{Z}_p = 10 \Omega$$

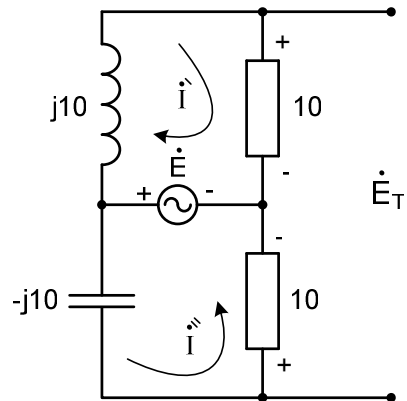
$$I_A = \left| \frac{\dot{E}_T}{\dot{Z}_T + \dot{Z}_p} \right| = \frac{400}{10 + 10} = 20 \text{ A} \rightarrow U_p = I_A \cdot Z_p = 20 \cdot 10 = 200 \text{ V}$$

$$I_1 = U_p \cdot Y_3 = 200 \cdot 0,05\sqrt{2} = 10\sqrt{2} \text{ A}$$

$$I_2 = U_p \cdot Y_4 = 200 \cdot 0,05\sqrt{2} = 10\sqrt{2} \text{ A}$$

$$\dot{I}' = \frac{\dot{E}}{10 + j10} = \frac{\dot{E}}{10\sqrt{2}|45^\circ} = 0,05\sqrt{2}|_{-45^\circ} \cdot \dot{E}$$

$$\dot{I}'' = \frac{\dot{E}}{10 - j10} = \frac{\dot{E}}{10\sqrt{2}|_{-45^\circ}} = 0,05\sqrt{2}|45^\circ \cdot \dot{E}$$



$$400 = \dot{I}' \cdot 10 - \dot{I}'' \cdot 10 = 10(\dot{I}' - \dot{I}'') = 10 \cdot 0,05\sqrt{2} \dot{E} (1|_{-45^\circ} - 1|45^\circ) =$$

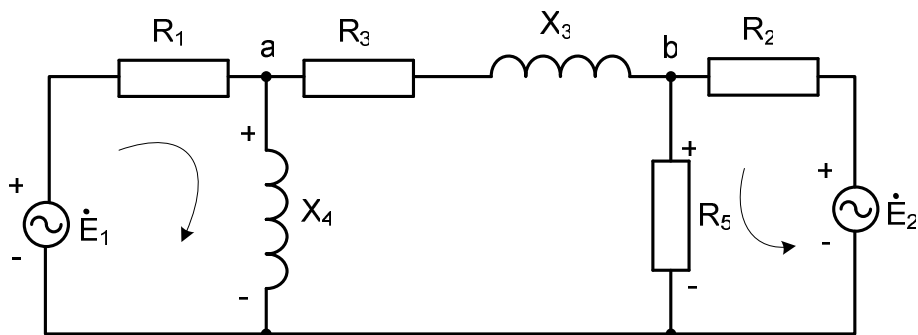
$$= 0,5\sqrt{2} \dot{E} (0,7071 - j0,7071 - 0,7071 - j0,7071) = 0,5\sqrt{2} \dot{E} \cdot 1,414|_{-90^\circ} = -j \dot{E}$$

$$\left| \dot{E} \right| = 400 \text{ V}$$

8.8. U shemi na slici zadano je: $R_1 = 5 \Omega$, $R_2 = 4 \Omega$, $R_3 = 3,1 \Omega$, $X_3 = 3,5 \Omega$, $X_4 = 5 \Omega$, $R_5 = 6 \Omega$,

$\dot{E}_1 = 30|0^\circ \text{ V}$, $\dot{E}_2 = 20|0^\circ \text{ V}$. Treba odrediti radnu snagu na otporu R_3 . Kolika bi morala

biti impedancija $\dot{Z}_3 = R_3 + jX_3$ da bi otpor R_3 trošio maksimalnu snagu? Kolika je ta snaga?



$$\begin{aligned} a) \quad \dot{Z}_T &= R_1 \parallel j4 + R_2 \parallel R_5 = \frac{5 \cdot j5}{5 + j5} + \frac{4 \cdot 6}{4 + 6} = \frac{25|90^\circ}{5\sqrt{2}|45^\circ} + \frac{24}{10} = 2,5\sqrt{2}|45^\circ + 2,4 = \\ &= 2,5 + j2,5 + 2,4 = 4,9 + j2,5 \Omega \end{aligned}$$

$$\begin{aligned} \dot{E}_T &= \frac{\dot{E}_1}{R_1 + jX_4} \cdot jX_4 - \frac{\dot{E}_2}{R_2 + R_5} \cdot R_5 = \frac{30}{5 + j5} j5 - \frac{20}{4 + 6} \cdot 6 = \frac{150|90^\circ}{5\sqrt{2}|45^\circ} - 12 = \\ &= 15\sqrt{2}|45^\circ - 12 = 15 + j15 - 12 = 3 + j15 \text{ V} \end{aligned}$$

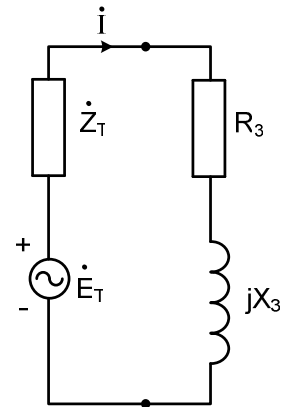
$$\dot{I} = \frac{\dot{E}_T}{\dot{Z}_T + R_3 + jX_3} = \frac{3 + j15}{4,9 + j2,5 + 3,1 + j3,5} = \frac{15,297 \angle 78,69^\circ}{10 \angle 36,87^\circ} = 1,5297 \angle 41,82^\circ \text{ A}$$

$$P_3 = I^2 R_3 = 1,5297^2 \cdot 3,1 = 7,254 \text{ W}$$

$$b) \quad \dot{Z}_3' = \dot{Z}_3^* = 4,9 - j2,5 \Omega$$

$$\begin{aligned} I_{\max} &= \frac{\dot{E}_T}{\dot{Z}_T + \dot{Z}_3'} = \frac{3 + j15}{4,9 + j2,5 + 4,9 - j2,5} \\ &= \frac{15,297 \angle 78,69^\circ}{2 \cdot 4,9} = 1,5609 \angle 78,69^\circ \text{ A} \end{aligned}$$

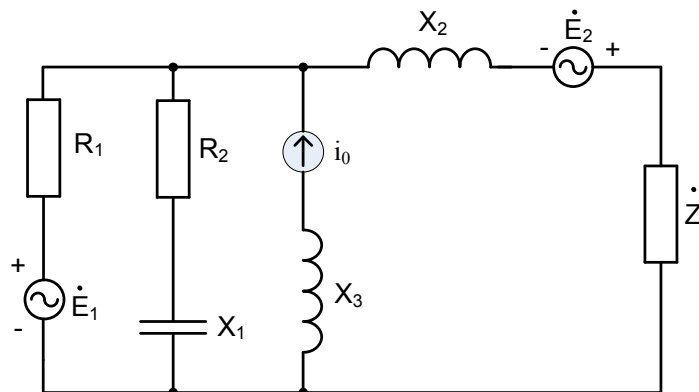
$$P_{3\max} = I_{\max}^2 \cdot R_{3\max} = 1,5609^2 \cdot 4,9 = 11,94 \text{ W}$$



8.9. U shemi na slici treba odrediti impedanciju \dot{Z} tako da se na njoj troši maksimalna snaga.

Kolika je ta snaga? Zadano: $R_1 = 20 \Omega$, $R_2 = 12 \Omega$, $X_1 = 16 \Omega$, $X_2 = 10 \Omega$, $X_3 = 16 \Omega$,

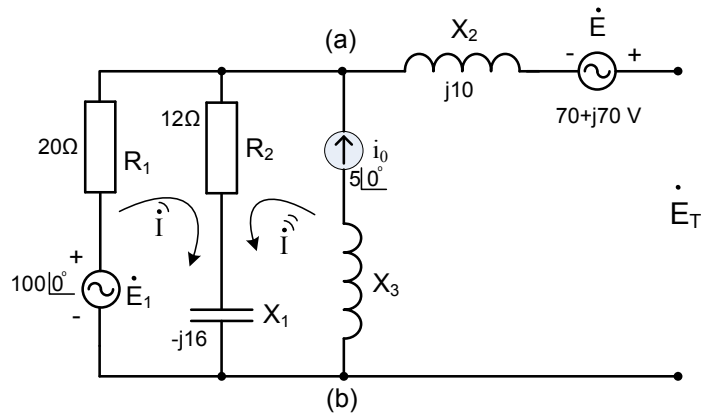
$$\dot{E}_1 = 100 \angle 0^\circ, \quad \dot{E}_2 = 70 + j70 \text{ V}, \quad \dot{I}_0 = 5 \angle 0^\circ.$$



$$\dot{Z}_T = \frac{20(12 - j16)}{20 + 12 - j16} + j10 = \frac{240 - j320}{32 - j16} + j10 = \frac{400 \angle -53,13^\circ}{35,7771 \angle -26,565^\circ} + j10 =$$

$$= 11,18 \angle -26,565^\circ + j10 = 10 - j5 + j10 = 10 + j5 \Omega$$

$$\dot{Z} = \dot{Z}_T^* = 10 - j5 \Omega$$



$$\begin{aligned} \dot{I}' &= \frac{100}{20+12-j16} = \frac{100}{35,7771 \angle -26,565^\circ} = 2,7951 \angle 26,565^\circ = 2,5 + j1,25 \text{ A} \\ \dot{I}'' &= 5 \frac{20(12-j16)}{20+12-j16} \cdot \frac{1}{12-j16} = \frac{100}{32-j16} = 2,5 + j1,25 \text{ A} \end{aligned} \quad \left. \vphantom{\begin{aligned} \dot{I}' &= \frac{100}{20+12-j16} \end{aligned}} \right\} \text{superpozicija}$$

$$\begin{aligned} \dot{E}_T &= (\dot{I}' + \dot{I}'')(12-j16) + 70 + j70 = (5 + j2,5)(12-j16) + 70 + j70 = \\ &= 100 - j50 + 70 + j70 = 170 + j20 = 171,172 \angle 6,71^\circ \text{ V} \end{aligned}$$

$$\dot{I} = \frac{\dot{E}_T}{\dot{Z}_T + \dot{Z}} = \frac{171,172 \angle 6,71^\circ}{20} = 8,5586 \angle 6,71^\circ \text{ A}$$

$$P = I^2 R = 8,5586^2 \cdot 10 = 732,5 \text{ W}$$

8.10. U shemi prema slici treba odrediti struju I primjenom a) Theveninova teorema, b) Nortonova teorema, c) teorema recipročnosti.

Zadano: $R_1 = R_4 = 2 \Omega$, $R_2 = 3 \Omega$, $X_2 = 4 \Omega$, $X_3 = 2 \Omega$, $X_4 = 3 \Omega$, $\dot{Z} = 5 + j \Omega$, $\dot{E} = 100 \angle -30^\circ \text{ V}$.

$$a) \quad \dot{Z}_1 = \frac{R_1 \cdot jX_3}{R_1 + jX_3} = \frac{2 \cdot j2}{2 + j2} = \frac{-j4}{2\sqrt{2} \angle 45^\circ} = \frac{2\sqrt{2}}{2} \angle 45^\circ = 1 + j \Omega$$

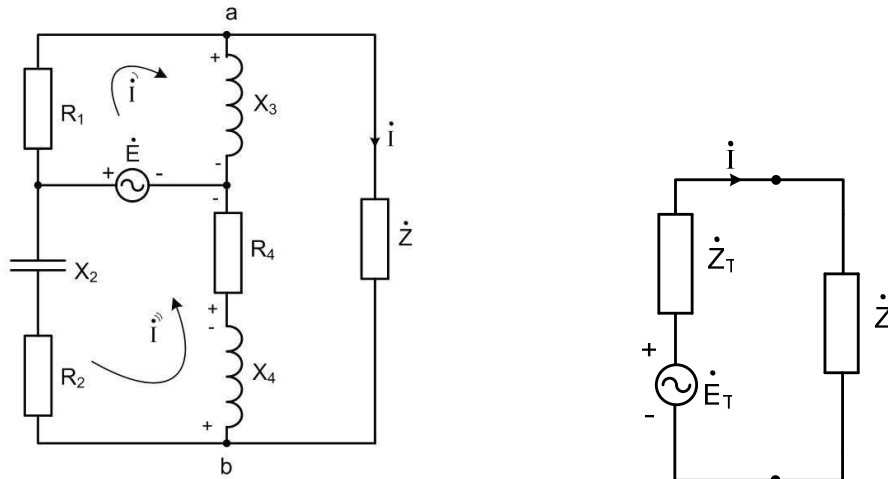
$$\dot{Y}_2 = \frac{1}{R_2 - jX_2} + \frac{1}{R_4 + jX_4} = \frac{1}{3 - j4} + \frac{1}{2 + j3} = \frac{1}{5 \angle -53,13^\circ} + \frac{1}{3,606 \angle 56,31^\circ} =$$

$$\begin{aligned} &= 0,2 \angle 53,13^\circ + 0,27735 \angle -56,31^\circ = 0,12 + j0,16 + 0,15385 - j0,23077 = 0,273846 - j0,07077 = \\ &= 0,282843 \angle -14,49^\circ \text{ S} \end{aligned}$$

$$\dot{Z}_2 = 3,5355 \angle 14,49^\circ = 3,423 + j0,8846 \Omega$$

$$\dot{Z}_T = \dot{Z}_1 + \dot{Z}_2 = 4,423 + j1,8846 = 4,80777 \angle 23,078^\circ \Omega$$

$$\dot{Z}_{uk} = \dot{Z}_T + \dot{Z} = 4,423 + j1,8846 + 5 + j = 9,4231 + j2,8846 = 9,8547 \angle 17,02^\circ \Omega$$



$$\dot{I}' = \frac{\dot{E}}{R_1 + jX_3} = \frac{100 \angle -30^\circ}{2 + j2} = \frac{100 \angle -30^\circ}{2\sqrt{2} \angle 45^\circ} = 25\sqrt{2} \angle -75^\circ \text{ A}$$

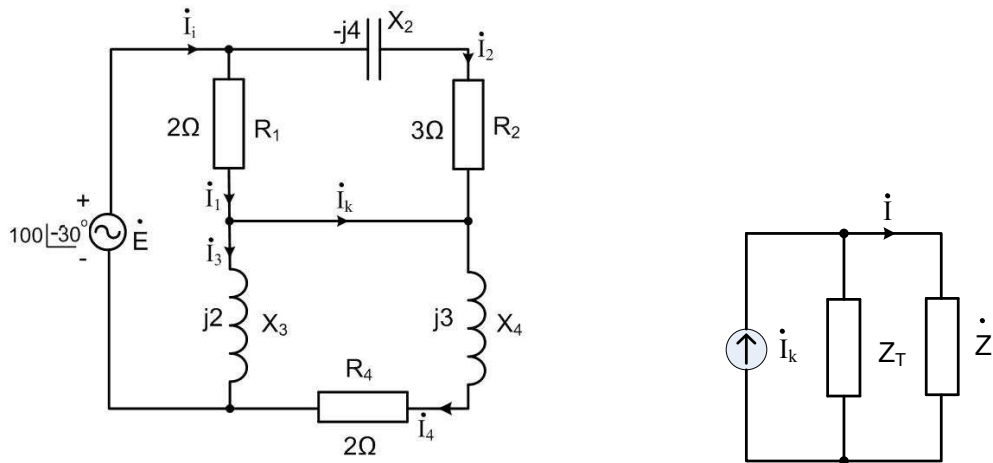
$$\dot{I}'' = \frac{\dot{E}}{R_2 + R_4 + j(X_4 - X_2)} = \frac{100 \angle -30^\circ}{3 + 2 + j(3 - 4)} = \frac{100 \angle -30^\circ}{5,099 \angle -11,31^\circ} = 19,612 \angle -18,69^\circ \text{ A}$$

$$\begin{aligned} \dot{E}_T &= \dot{I}' jX_3 - \dot{I}''(R_4 + jX_4) = 25\sqrt{2} \angle -75^\circ \cdot 2 \angle 90^\circ - 19,612 \angle -18,69^\circ \cdot 3,60555 \angle 56,31^\circ = \\ &= 50\sqrt{2} \angle 15^\circ - 70,71 \angle 37,62^\circ = 68,3 + j18,3 - 56,008 - j43,1632 = 12,3 - j24,862 = \\ &= 27,735 \angle -63,69^\circ \text{ V} \end{aligned}$$

$$\dot{I} = \frac{\dot{E}_T}{\dot{Z}_{uk}} = \frac{27,735 \angle -63,69^\circ}{9,8547 \angle 17,02^\circ} = 2,8144 \angle -80,71^\circ \text{ A}$$

$$\begin{aligned} b) \quad \dot{Y}_1 &= \frac{1}{2} + \frac{1}{3 - j4} = 0,5 + \frac{1}{5 \angle -53,13^\circ} = 0,5 + 0,2 \angle 53,13^\circ = 0,5 + 0,12 + j0,16 = 0,62 + j0,16 = \\ &= 0,64031 \angle 14,47^\circ \text{ S} \end{aligned}$$

$$\begin{aligned} \dot{Y}_2 &= \frac{-j}{2} + \frac{1}{2 + j3} = -0,5j + \frac{1}{3,60555 \angle 56,31^\circ} = -0,5j + 0,27735 \angle -56,31^\circ = \\ &= -0,5j + 0,153846 - j0,23077 = 0,153846 - j0,73077 = 0,746789 \angle -78,111^\circ \text{ S} \end{aligned}$$



$$\dot{Z}_1 = \frac{1}{\dot{Y}_1} = \frac{1}{0,64031 \angle 14,47^\circ} = 1,56174 \angle -14,47^\circ = 1,5122 - j0,39024 \Omega$$

$$\dot{Z}_2 = \frac{1}{\dot{Y}_2} = \frac{1}{0,746789 \angle -78,111^\circ} = 1,339066 \angle 78,111^\circ = 0,27587 + j1,31034 \Omega$$

$$\dot{Z}_4' = \dot{Z}_1 + \dot{Z}_2 = 1,78807 + j0,9201 = 2,0109 \angle 27,23^\circ$$

$$\dot{I}_i = \frac{\dot{E}}{\dot{Z}_4'} = \frac{100 \angle -30^\circ}{2,0109 \angle 27,23^\circ} = 49,7285 \angle -57,23^\circ \text{ A}$$

$$\dot{U}_1 = \dot{I}_i \cdot \dot{Z}_1 = 49,7285 \angle -57,23^\circ \cdot 1,56174 \angle -14,47^\circ = 77,663 \angle -71,7^\circ \text{ V}$$

$$\dot{U}_2 = \dot{I}_i \cdot \dot{Z}_2 = 49,7285 \angle -57,23^\circ \cdot 1,339066 \angle 78,111^\circ = 66,589 \angle 20,88^\circ \text{ V}$$

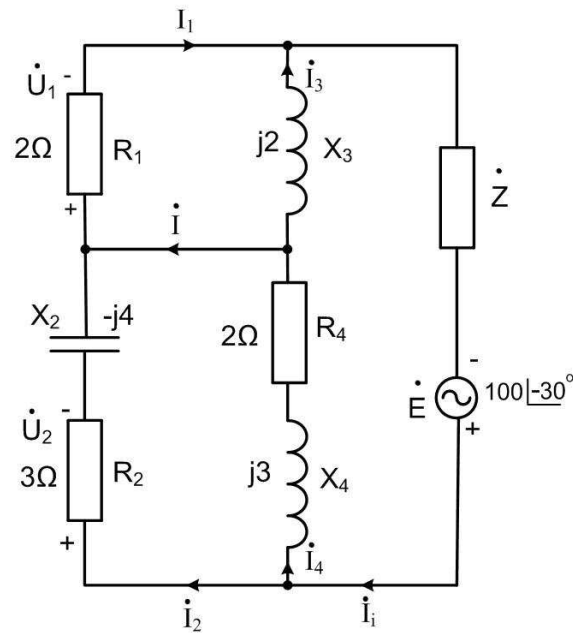
$$\dot{I}_2 = \frac{\dot{U}_1}{R_2 - jX_2} = \frac{77,663 \angle -71,7^\circ}{5 \angle -53,13^\circ} = 15,5326 \angle -18,57^\circ = 14,723955 - j4,9464 \text{ A}$$

$$\dot{I}_4 = \frac{\dot{U}_2}{R_4 + jX_4} = \frac{66,5898 \angle 20,88^\circ}{3,606 \angle 56,31^\circ} = 18,46638 \angle -35,43^\circ = 15,047159 - j10,704681 \text{ A}$$

$$\begin{aligned} \dot{I}_k &= -\dot{I}_2 + \dot{I}_4 = -14,723955 + j4,9464 + 15,047159 - j10,704681 = 0,3232 - j5,75828 = \\ &= 5,7673 \angle -86,79^\circ \text{ A} \end{aligned}$$

$$\dot{I} = \dot{I}_k \cdot \frac{\dot{Z}_T \cdot \dot{Z}}{\dot{Z}_T + \dot{Z}} \cdot \frac{1}{\dot{Z}} = 5,7673 \angle -86,79^\circ \cdot \frac{4,80777 \angle 23,08^\circ}{9,8547 \angle 17,02^\circ} = 2,814 \angle -80,73^\circ \text{ A}$$

c)



$$a) \rightarrow \dot{Z}_{uk} = 9,8547 \angle 17,02^\circ \rightarrow \dot{I}_i = \frac{\dot{E}}{\dot{Z}_{uk}} = \frac{100 \angle -30^\circ}{9,8547 \angle 17,02^\circ} = 10,14744 \angle -47,02^\circ \text{ A}$$

$$\dot{U}_1 = \dot{I}_i \cdot \dot{Z}_1 = 10,14744 \angle -47,02^\circ \cdot \sqrt{2} \angle 45^\circ = 14,3506 \angle -2,02^\circ \text{ V}$$

$$\dot{U}_2 = \dot{I}_i \cdot \dot{Z}_2 = 10,14744 \angle -47,02^\circ \cdot 3,5355 \angle 14,49^\circ = 35,8763 \angle -32,53^\circ \text{ V}$$

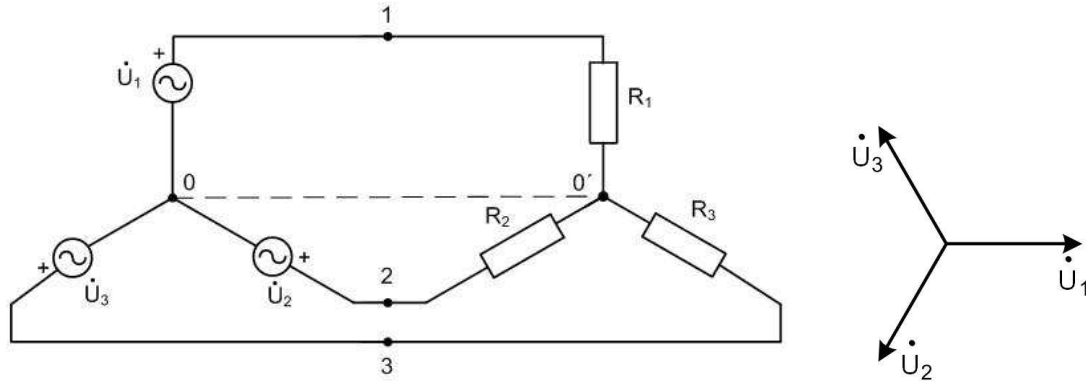
$$\dot{I}_4 = \frac{\dot{U}_2}{R_4 + jX_4} = \frac{35,8763 \angle -32,53^\circ}{3,606 \angle -56,31^\circ} = 9,949 \angle -88,84^\circ = 0,201413 - j9,947 \text{ A}$$

$$\dot{I}_3 = \frac{\dot{U}_1}{jX_3} = \frac{14,3506 \angle -2,02^\circ}{2 \angle 90^\circ} = 7,1753 \angle -92,02^\circ = -0,25292 - j7,1709 \text{ A}$$

$$\dot{I} = \dot{I}_4 - \dot{I}_3 = 0,201413 - j9,947 + 0,25292 + j7,1709 = 0,45433 - j2,7761 = 2,813 \angle -80,71^\circ \text{ A}$$

9. Višefazni sustav izmjeničnih struja

9.1. Treba odrediti sve struje i snagu trofaznog trošila spojenog u zvijezdu a) s nulvodičem ($R_0 = 0$) i b) bez nulvodiča. Zadano: otpori faza $R_1 = 50 \Omega$; $R_2 = R_3 = 100 \Omega$ i fazni napon $\dot{U}_{f1} = 220 \angle 0^\circ \text{ V}$ simetričnog trofaznog sustava EMS.



a) $R_0 = 0$ $\dot{I}_1 = \frac{\dot{U}_1}{R_1} = \frac{220 \angle 0^\circ}{50} = 4,4 \angle 0^\circ \text{ A}$

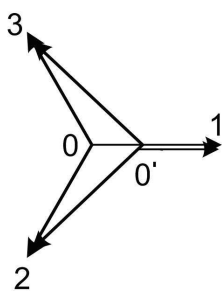
$$\dot{I}_2 = \frac{\dot{U}_2}{R_2} = \frac{220 \angle -120^\circ}{100} = 2,2 \angle -120^\circ = -1,1 - j1,905 \text{ A}$$

$$\dot{I}_3 = \frac{\dot{U}_3}{R_3} = \frac{220 \angle 120^\circ}{100} = 2,2 \angle 120^\circ = -1,1 + j1,905 \text{ A}$$

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 4,4 - 1,1 - j1,905 - 1,1 + j1,905 = 2,2 \angle 0^\circ \text{ A}$$

$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 = 4,4^2 \cdot 50 + 2 \cdot 2,2^2 \cdot 100 = 1936 \text{ W}$$

b) $R_0 = \infty$ $\dot{U}_{0'0} = \frac{\frac{220}{50} + \frac{220 \angle -120^\circ}{100} + \frac{220 \angle 120^\circ}{100}}{\frac{1}{50} + \frac{1}{100} + \frac{1}{100}} = \frac{2,2 \angle 0^\circ}{0,04} = 55 \angle 0^\circ \text{ V}$



$$\dot{U}_{10'} = \dot{U}_{10} - \dot{U}_{0'0} = 220 \angle 0^\circ - 55 = 165 \text{ V} \rightarrow \dot{I}_1 = 3,3 \text{ A}$$

$$\begin{aligned} \dot{U}_{20'} &= \dot{U}_{20} - \dot{U}_{0'0} = 220 \angle -120^\circ - 55 = -110 - j190,5 - 55 \\ &= -165 - j190,5 = 252,04 \angle -130,88^\circ \text{ V} \end{aligned}$$

$$\dot{I}_2 = \frac{\dot{U}_{20'}}{R_2} = \frac{252,04 \angle -130^\circ}{100} = 2,52 \angle -130,1^\circ \text{ A}$$

$$\dot{U}_{30'} = \dot{U}_{30} - \dot{U}_{0'0} = 220 \angle 120^\circ - 55 = -110 + j190,5 - 55 = 252,04 \angle 130,88^\circ \text{ V}$$

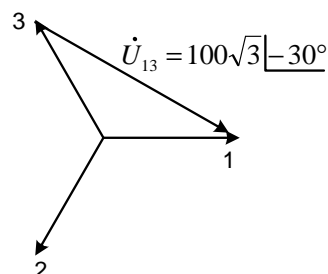
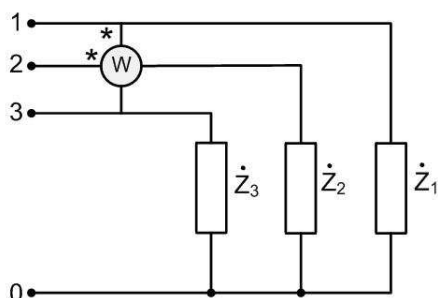
$$\dot{I}_3 = \frac{\dot{U}_{30'}}{R_3} = 2,52 \angle 131^\circ \text{ A}$$

$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3 = 3,3^2 \cdot 50 + 2,52^2 \cdot 100 \cdot 2 = 1815 \text{ W}$$

9.2. Trofazno trošilo prema slici spojeno je na simetrični sustav EMS faznog napona

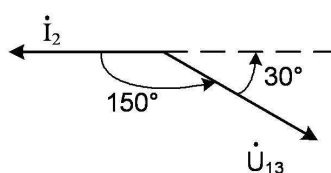
$U_f = 100 \text{ V}$. Treba odrediti pokazivanje vatmetra ako je $\dot{Z}_1 = 10 \angle 90^\circ \Omega$,

$\dot{Z}_2 = 10 \angle 60^\circ \Omega$, $\dot{Z}_3 = 10 \angle 30^\circ \Omega$.



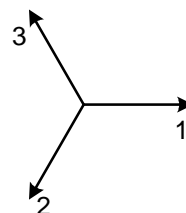
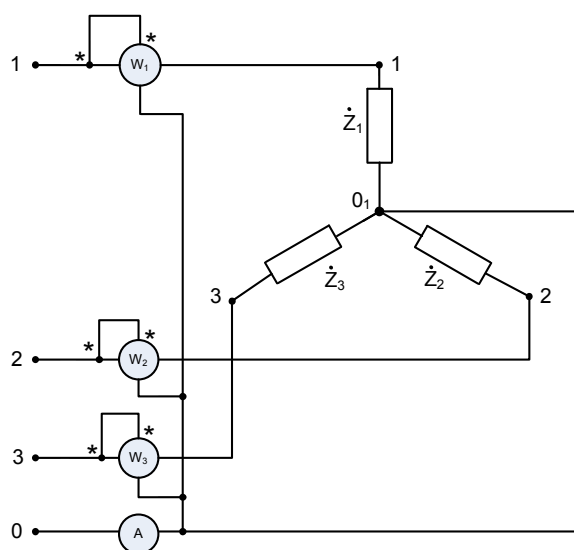
$$\dot{I}_2 = \frac{\dot{U}_2}{\dot{Z}_2} = \frac{100 \angle -120^\circ}{10 \angle 60^\circ} = 10 \angle -180^\circ \text{ A}$$

$$P_W = U_{13} I_2 \cos \alpha = 100\sqrt{3} \cdot 10 \cdot \cos 150^\circ = -1500 \text{ W}$$



9.3. U shemi na slici treba odrediti što pokazuju instrumenti ako je: $\dot{Z}_1 = 3 + j4 \Omega$,

$\dot{Z}_2 = 6 + j8 \Omega$, $\dot{Z}_3 = 12 + j16 \Omega$ i linijski napon $U_l = 173 \text{ V}$.



$$\dot{I}_1 = \frac{\dot{U}_1}{\dot{Z}_1} = \frac{100}{5 \angle 53,13^\circ} = 20 \angle -53,13^\circ = 12 - j16 \text{ A}$$

$$P_1 = I_1^2 R_1 = 20^2 \cdot 3 = 1200 \text{ W}$$

$$\dot{I}_2 = \frac{\dot{U}_2}{\dot{Z}_2} = \frac{100 \angle -120^\circ}{10 \angle 53,13^\circ} = 10 \angle -173,13^\circ = -9,982 - j1,19615 \text{ A}$$

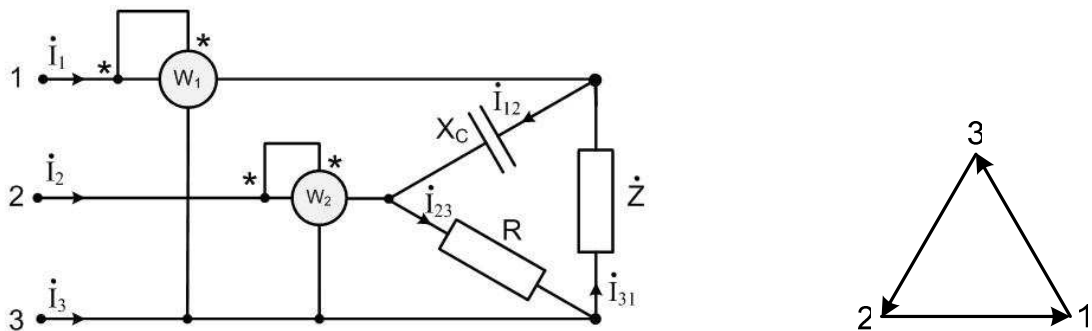
$$P_2 = I_2^2 R_2 = 10^2 \cdot 6 = 600 \text{ W}$$

$$\dot{I}_3 = \frac{\dot{U}_3}{\dot{Z}_3} = \frac{100 \angle 120^\circ}{20 \angle 53,13^\circ} = 5 \angle 66,87^\circ = 1,9641 + j4,598 \text{ A}$$

$$P_3 = I_3^2 R_3 = 5^2 \cdot 12 = 300 \text{ W}$$

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 4,036 - j12,598 = 13,23 \angle -72,2^\circ \text{ A} \rightarrow I_0 = 13,23 \text{ A}$$

9.4. U shemi na slici zadano je: $R = 30 \, \Omega$, $X_C = 30 \, \Omega$, $\dot{Z} = 30 \angle 60^\circ \, \Omega$ i linijski napon $U_l = 420 \, \text{V}$. Treba odrediti sve struje.



$$\dot{I}_{12} = \frac{\dot{U}_{12}}{-jX_C} = \frac{420 \angle 0^\circ}{30 \angle -90^\circ} = 14 \angle 90^\circ = j14 \, \text{A}$$

$$\dot{I}_{23} = \frac{\dot{U}_{23}}{R} = \frac{420 \angle -120^\circ}{30} = 14 \angle -120^\circ = -7 - j12,124 \, \text{A}$$

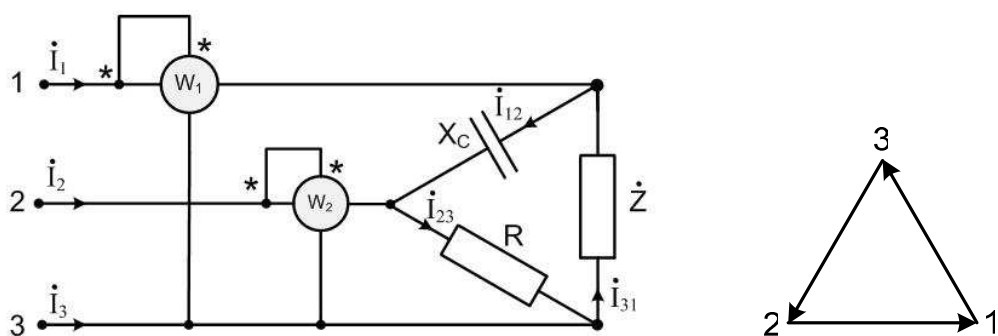
$$\dot{I}_{31} = \frac{\dot{U}_{31}}{\dot{Z}} = \frac{420 \angle 120^\circ}{30 \angle 60^\circ} = 14 \angle 60^\circ = 7 + j12,124 \, \text{A}$$

$$\dot{I}_1 = \dot{I}_{12} - \dot{I}_{31} = j14 - 7 - j12,124 = 7,247 \angle 165^\circ \, \text{A}$$

$$\dot{I}_2 = \dot{I}_{23} - \dot{I}_{12} = -7 - j12,124 - j14 = 27,046 \angle 255^\circ \, \text{A}$$

$$\dot{I}_3 = \dot{I}_{31} - \dot{I}_{23} = 7 + j12,124 + 7 + j12,124 = 28 \angle 60^\circ \, \text{A}$$

9.5. U shemi na slici treba odrediti što pokazuju vatmetri i izračunati ukupnu radnu snagu sustava. Zadano: $R = 30 \, \Omega$, $X_C = 30 \, \Omega$, $\dot{Z} = 30 \angle 60^\circ \, \Omega$ i linijski napon $U_l = 420 \, \text{V}$.

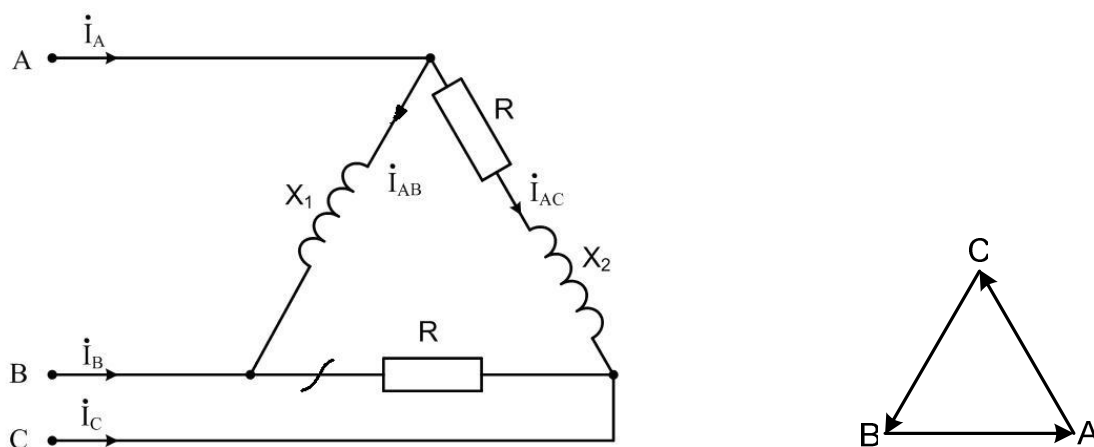


$$P_{W1} = U_{13} \cdot I_1 \cdot \cos \alpha = 420 \cdot 7,247 \cos(-60^\circ - 165^\circ) = -2152 \, \text{W}$$

$$P_{W2} = U_{23} \cdot I_2 \cdot \cos \beta = 420 \cdot 27,046 \cos(-120^\circ - 255^\circ) = 10972 \, \text{W}$$

$$P = P_{W1} + P_{W2} = 8820 \, \text{W}$$

9.6. Na nesimetričnom trofaznom trošilu, prema slici, koje je priključeno na simetrični generator $U_l = 380$ V došlo je do prekida u fazi CB. Ako su u tom slučaju linijske struje $I_B = I_C = 1$ A, a snaga na trošilu iznosi $P = 100$ W, treba odrediti R , X_1 , X_2 .



$$-\dot{I}_B = \dot{I}_{AB} = \frac{\dot{U}_{AB}}{X_1 \angle 90^\circ} = \frac{380 \angle 0^\circ}{X_1 \angle 90^\circ} = 1 \angle -90^\circ \text{ A} \rightarrow X_1 = 380 \Omega$$

$$P = I_{AC}^2 R \rightarrow R = \frac{P}{I_{AC}^2} = \frac{100}{1} = 100 \Omega$$

$$-\dot{I}_C = \dot{I}_{AC} = \frac{\dot{U}_{AC}}{R + jX_2} = \frac{380 \angle -60^\circ}{R + jX_2} = 1 \angle \varphi_{AC} \text{ A}$$

$$Z_{AC} = 380 \Omega = \sqrt{R^2 + X_2^2} \rightarrow X_2 = \sqrt{Z_{AC}^2 - R^2} = \sqrt{380^2 - 100^2} = 366,6 \Omega$$

$$\varphi_2 = \arctg \frac{X_2}{R} = 74,74^\circ$$

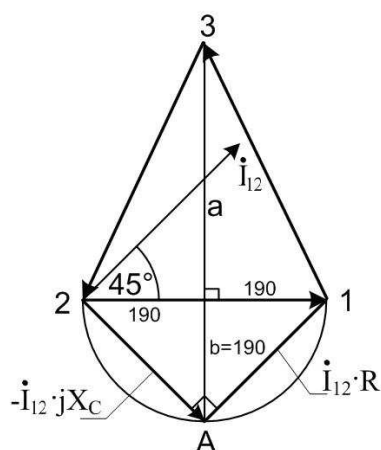
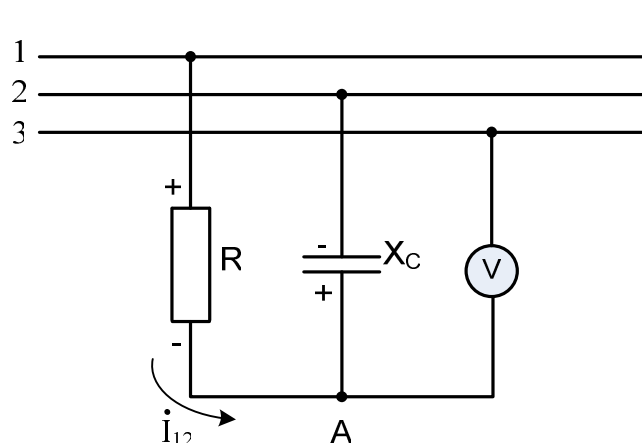
$$\varphi_{AC} = -60^\circ - \varphi_2 = -60^\circ - 74,74^\circ = -134,74^\circ$$

možemo izračunati struje:

$$-\dot{I}_C = 1 \angle -60^\circ - 74,74^\circ = 1 \angle -134,74^\circ = -0,704 - j0,71 \text{ A}$$

$$\dot{I}_A = -\dot{I}_B - \dot{I}_C = -j - 0,704 - j0,71 = -0,704 - j1,71 = 1,849 \angle -112,4^\circ \text{ A}$$

9.7. Koji napon pokazuje voltmetar na slici? Zadano: $U_l = 380 \text{ V}$, $R = X_C$.



$$\operatorname{tg} \varphi = \frac{-X}{R} = -1 \rightarrow \varphi = -45^\circ$$

$$Z_{12} = \sqrt{2R^2} = R\sqrt{2}$$

$$\dot{I}_{12} = \frac{\dot{U}_{12}}{Z_{12}} = \frac{380 \angle 0^\circ}{R\sqrt{2} \angle -45^\circ}$$

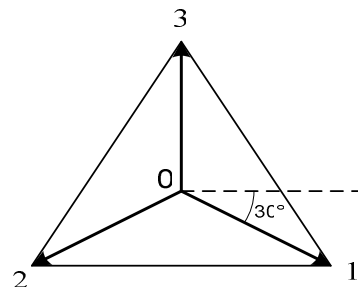
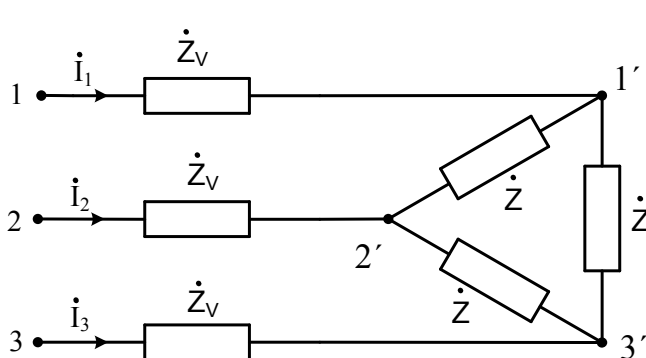
$$a = \sqrt{380^2 - 190^2} = 329 \text{ V}$$

$$U_{A3} = a + b = 519,1 \text{ V}$$

9.8. Treba odrediti linijske i fazne struje, te fazne napone na trošilu prema slici, koje je

priključeno na simetrični sustav EMS $U_l = 230 \text{ V}$. Zadano: $\dot{Z}_V = 0,5 + j0,5 \Omega$,

$\dot{Z} = 8,4 + j6,6 \Omega$, $\dot{U}_{12} = 230 \angle 0^\circ \text{ V}$.



$$\dot{Z}_V = 0,5\sqrt{2} \angle 45^\circ = 0,7071 \angle 45^\circ$$

$$\dot{Z}' = \frac{\dot{Z}}{3} = 2,8 + j2,2 \rightarrow \dot{Z}_f = \dot{Z}_V + \dot{Z}' = 0,5 + j0,5 + 2,8 + j2,2 = 3,3 + j2,7 = 4,2638 \angle 39,2894^\circ \Omega$$

$$\dot{I}_1 = \frac{\dot{U}_1}{\dot{Z}_f} = \frac{230 \angle -30^\circ}{\sqrt{3} \cdot 4,2638 \angle 39,2894^\circ} = 31,144 \angle -69,2894^\circ \text{ A}$$

$$\dot{I}_2 = \frac{\dot{U}_2}{\dot{Z}_f} = \frac{132,79 \angle -150^\circ}{4,2638 \angle 39,2894^\circ} = 31,144 \angle -189,2894^\circ \text{ A}$$

$$\dot{I}_3 = \frac{\dot{U}_3}{\dot{Z}_f} = \frac{132,79 \angle 90^\circ}{4,2638 \angle 39,2894^\circ} = 31,144 \angle 50,7106^\circ \text{ A}$$

$$\dot{\phi}_1' = \dot{U}_1 - \dot{I}_1 \dot{Z}_V = 132,79 \angle -30^\circ - 31,144 \angle -69,3^\circ \cdot 0,7071 \angle 45^\circ = 94,9277 - j57,334 \text{ V}$$

$$\dot{\phi}_2' = \dot{U}_2 - \dot{I}_2 \dot{Z}_V = 132,79 \angle -150^\circ - 31,144 \angle -189,3^\circ \cdot 0,7071 \angle 45^\circ = -97,119 - j53,5413 \text{ V}$$

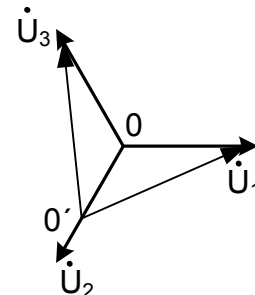
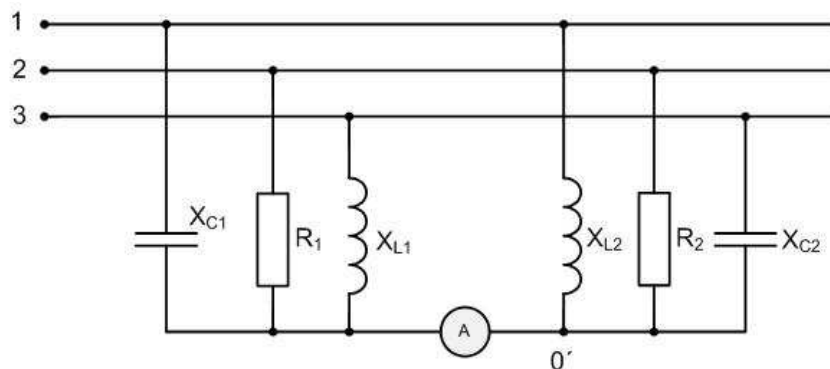
$$\dot{\phi}_3' = \dot{U}_3 - \dot{I}_3 \dot{Z}_V = 132,79 \angle 90^\circ - 31,144 \angle 50,71^\circ \cdot 0,7071 \angle 45^\circ = 2,191 + j110,878 \text{ V}$$

$$\dot{U}_{1'2'} = \dot{\phi}_1' - \dot{\phi}_2' = 192,08 \angle -1,131^\circ \text{ V} \rightarrow \dot{I}_{1'2'} = \frac{\dot{U}_{1'2'}}{\dot{Z}} = \frac{192,08 \angle -1,131^\circ}{10,683 \angle 38,157^\circ} = 17,98 \angle -39,29^\circ \text{ A}$$

$$\dot{U}_{2'3'} = \dot{\phi}_2' - \dot{\phi}_3' = 192,08 \angle -121,13^\circ \text{ V} \rightarrow \dot{I}_{2'3'} = \frac{\dot{U}_{2'3'}}{\dot{Z}} = \frac{192,08 \angle -121,13^\circ}{10,683 \angle 38,157^\circ} = 17,98 \angle 200,71^\circ \text{ A}$$

$$\dot{U}_{3'1'} = \dot{\phi}_3' - \dot{\phi}_1' = 192,08 \angle 118,87^\circ \text{ V} \rightarrow \dot{I}_{3'1'} = \frac{\dot{U}_{3'1'}}{\dot{Z}} = \frac{192,08 \angle 118,87^\circ}{10,683 \angle 38,157^\circ} = 17,98 \angle 80,71^\circ \text{ A}$$

9.9. Treba odrediti linijski napon simetričnog izvora EMS ako idealni ampermetar u shemi na slici pokazuje $I_A = 2,7 \text{ A}$. Zadano: $R_1 = 10 \Omega$, $X_{L1} = X_{C1} = 50 \Omega$, $R_2 = X_{L2} = X_{C2} = 100 \Omega$.



$$\dot{U}_1 = U \angle 0^\circ$$

$$\dot{U}_2 = U \angle -120^\circ$$

$$\dot{U}_3 = U \angle 120^\circ$$

$$\dot{Z}_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \cdot 100}{100 + 100} = \frac{100}{11} \Omega$$

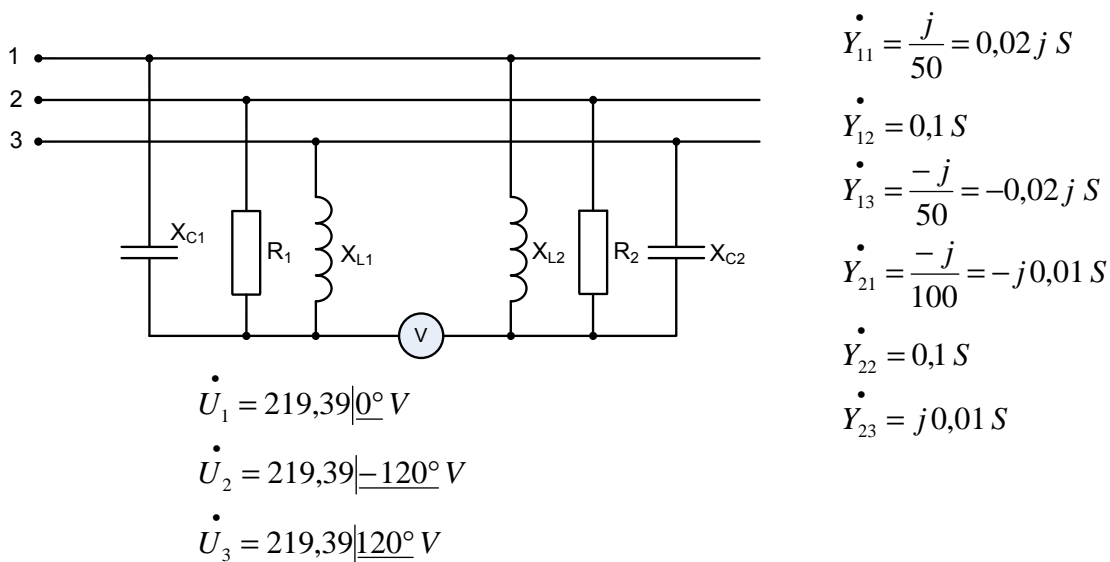
$$\dot{Z}_A = \frac{-j100 \cdot j50}{j100 - j50} = -j100 = 100 \angle -90^\circ \Omega$$

$$\dot{Z}_C = \frac{-j50 \cdot j100}{-j100 + j50} = j100 \Omega$$

$$\dot{U}_{0'0} = \frac{U \cdot 0,01 \angle 90^\circ + U \angle -120^\circ \cdot \frac{11}{100} + U \angle 120^\circ \cdot 0,01 \angle -90^\circ}{j0,01 + \frac{11}{100} - j0,01} =$$

$$\begin{aligned}
&= \frac{100}{11} U (j0,01 - 0,055 - j0,0956 + 8,66 \cdot 10^{-3} + j0,005) = U \cdot 0,84254 \angle -120^\circ = \\
&= U (-0,42127 - j0,72966) \\
\dot{U}_{10'} &= \dot{U}_1 - \dot{U}_{0'0} = U - U (-0,42127 - j0,72966) = U (1,42127 + j0,72966) = U \cdot 1,5976 \angle 27,175^\circ V \\
\dot{U}_{20'} &= \dot{U}_2 - \dot{U}_{0'0} = U \angle -120^\circ - U \cdot 0,84254 \angle -120^\circ = U \cdot 0,15746 \angle -120^\circ V \\
\dot{U}_{30'} &= \dot{U}_3 - \dot{U}_{0'0} = U (-0,5 + j0,86602) - U (-0,42127 - j0,72966) = U (-0,07873 + j1,595687) = \\
&= U \cdot 1,5976 \angle 92,825^\circ V \\
\dot{I}_1' &= \frac{U \cdot 1,6 \angle 27,18^\circ}{50 \angle -90^\circ} = U \cdot 0,032 \angle 117,18^\circ = U (-0,01458 + j0,02839) \\
\dot{I}_2' &= \frac{U \cdot 0,15746 \angle -120^\circ}{10} = U \cdot 0,015746 \angle -120^\circ = U (-0,007873 - j0,013636) \\
\dot{I}_3' &= \frac{U \cdot 1,6 \angle 92,825^\circ}{50 \angle 90^\circ} = U \cdot 0,032 \angle 2,825^\circ = U (0,031913 + j0,0015747) \\
\dot{I}_1' + \dot{I}_2' + \dot{I}_3' &= U [0,0094624 + j0,016328] = U \cdot 0,01887 \angle 59,91^\circ \rightarrow 2,7 A \\
U &= \frac{2,7}{0,018871} = 143,07 V \rightarrow U_C = \sqrt{3}U = 247,8 V
\end{aligned}$$

9.10. Ako se u shemi na slici umjesto ampermetra stavi idealni voltmetar, treba odrediti njegovo pokazivanje, ako je $U_l = 380 V$, a ostali su podaci dani u zadatku 9.9.

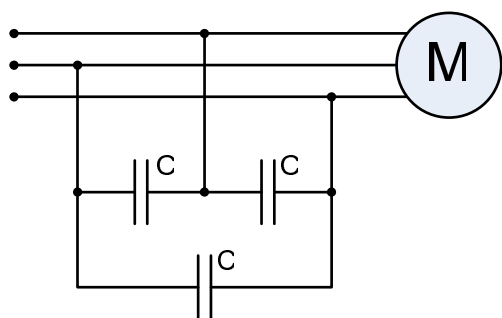


$$\begin{aligned}
\dot{U}_{0'0} &= 219,39 \cdot \frac{1 \cdot 0,02 j + 1 \angle -120^\circ \cdot 0,1 + 1 \angle 120^\circ \cdot 0,02 \angle -90^\circ}{0,02 j + 0,1 - j0,02} = \\
&= 219,39 (0,2 j - 0,5 - j0,866 + 0,1732 + 0,1) = 219,39 (-0,3268 - j0,566) = \\
&= 219,39 \cdot 0,6539 \angle -120^\circ = 143,39 \angle -120^\circ V
\end{aligned}$$

$$\begin{aligned}\dot{U}_{0'0} &= 219,39 \frac{-j0,01 + 1 \angle -120^\circ \cdot 0,01 + 1 \angle 120^\circ \cdot 0,01 \angle 90^\circ}{-j0,01 + 0,01 + j0,01} = \\ &= 219,39(-j - 0,05 - j0,866 - 0,866 - j0,5) = 219,39(-1,366 - j2,366) = \\ &= 219,39 \cdot 2,732 \angle -120^\circ = 599,39 \angle -120^\circ \text{ V}\end{aligned}$$

$$\dot{U}_V = \dot{U}_{0'0} - \dot{U}_{00} = 599,39 \angle -120^\circ - 143,39 \angle -120^\circ = 456 \angle -120^\circ \text{ V}$$

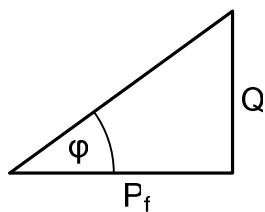
9.11. Trofazni simetrični asinkroni motor snage $P = 3,5 \text{ kW}$ i $\cos \varphi = 0,6$ priključen je na trofaznu mrežu $U_l = 220 \text{ V}$ i $f = 50 \text{ Hz}$. Treba odrediti kapacitet C prema slici, tako da se faktor snage povisi na $\cos \varphi' = 0,8$.



$$P_f = \frac{P}{3} = 1166,6 \text{ W}$$

$$\cos \varphi = 0,6 \rightarrow \operatorname{tg} \varphi = 1,3$$

$$\cos \varphi' = 0,8 \rightarrow \operatorname{tg} \varphi' = 0,75$$

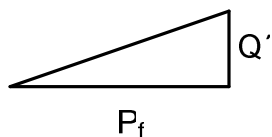


$$\frac{Q}{P_f} = \operatorname{tg} \varphi$$

$$Q = P_f \cdot \operatorname{tg} \varphi = 1555,5 \text{ VAR}$$

$$\operatorname{tg} \varphi' = \frac{Q - Q_c}{P_f}$$

$$Q - Q_c = P_f \cdot \operatorname{tg} \varphi' = 875$$

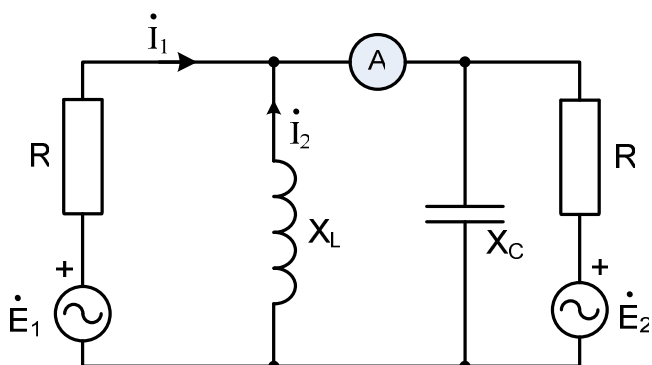
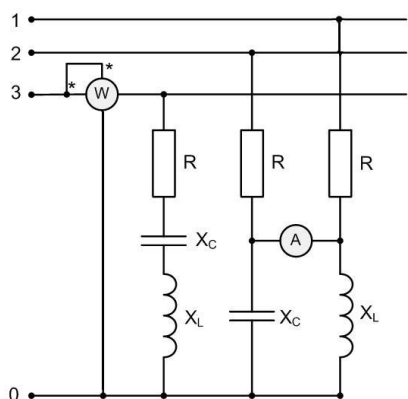


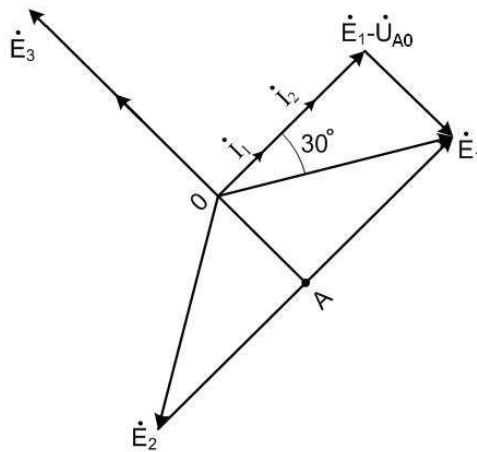
$$Q_c = Q - 875 = 1555,5 - 875 = 680,5 \text{ VAR}$$

$$Q_c = \frac{U_l^2}{X_c} \rightarrow X_c = \frac{U_l^2}{Q_c} = \frac{220^2}{680,5} = 71,118 = \frac{1}{\omega C}$$

$$C = \frac{1}{71,118 \omega} = 44,76 \mu\text{F}$$

9.12. Treba odrediti snagu koju mjeri vatmetar u shemi na slici, ako ampermetar pokazuje $I_A = 20 \text{ A}$, a $X_C = X_L = \sqrt{3} \Omega$, $R = 10\sqrt{3} \Omega$. Izvor je simetričan.





$$\dot{U}_{A0} = \frac{\frac{\dot{E}_1}{R} + \frac{\dot{E}_2}{R}}{\frac{1}{R} + \frac{j}{X_C} + \frac{1}{R} - \frac{j}{X_L}} = \frac{\frac{1}{R} [U_f | 0^\circ + U_f | -120^\circ]}{\frac{2}{R}} = -\frac{\dot{E}_3}{2} = \frac{U_f}{2} | -60^\circ$$

$$\dot{I}_1 = \frac{\dot{E}_1 - \dot{U}_{A0}}{R} = \frac{U_f | 0^\circ + \frac{U_f}{2} | 120^\circ}{R} = \frac{U_f}{R} [1 + 0,5 | 120^\circ] = \frac{U_f}{10\sqrt{3}} \cdot \frac{\sqrt{3}}{2} | 30^\circ = \frac{U_f}{20} | 30^\circ \text{ A}$$

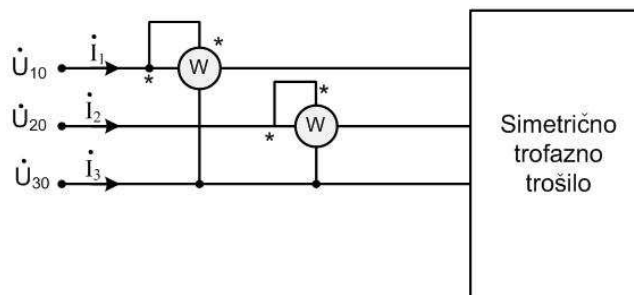
$$\dot{I}_2 = \frac{-\dot{U}_{A0}}{jX_L} = \frac{\frac{U_f}{2} | 120^\circ}{\sqrt{3} | 90^\circ} = \frac{U_f}{2\sqrt{3}} | 30^\circ \text{ A}$$

$$\dot{I}_A = \dot{I}_1 + \dot{I}_2 = \left[\frac{U_f}{20} + \frac{U_f}{2\sqrt{3}} \right] | 30^\circ$$

$$U_f \cdot 0,338675 = 20$$

$$U_f = 59,054 \text{ V} \rightarrow P_W = \frac{U_f^2}{R} = \frac{59,054^2}{10\sqrt{3}} = 201,34 \text{ W}$$

9.13. Metodom dvaju vatmetara na simetričnom trošilu prema slici, izmjereno je $P_1 = 100 \text{ W}$ i $P_2 = 500 \text{ W}$. Treba odrediti: a) snagu trofaznog trošila, b) faktor snage.



$$a) \quad P = P_1 + P_2 = 600 \text{ W}$$

$$\varphi = -49,11^\circ$$

$$b) \quad \tan \varphi = \sqrt{3} \cdot \frac{P_1 - P_2}{P_1 + P_2} = \sqrt{3} \frac{100 - 500}{100 + 500}$$

$$\cos \varphi = 0,655$$

10. Proračun mreža s nesinusnim periodičkim valnim oblicima

10.1 Serijski spoj RC priključen je na napon koji sadrži prvi, treći i peti harmonik, čija je efektivna vrijednost $U = 120$ V. Treba odrediti RC elemente ako je: $P = 175$ W, $\omega = 3140$ rad/s, $i = 5\sin\omega t + 3\sin 3\omega t + \sin 5\omega t$ A.

$$I^2 = \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{25+9+1}{2} = 17,5 = \frac{P}{R} \Rightarrow R = \frac{P}{I^2} = \frac{175}{17,5} = 10 \Omega$$

$$X_1 = \frac{1}{\omega C} \rightarrow X_3 = \frac{X_1}{3} ; X_5 = \frac{X_1}{5}$$

$$\left. \begin{aligned} U_1 &= I_1 Z_1 = I_1 \sqrt{R^2 + X_1^2} / ^2 \\ U_3 &= I_3 Z_3 = I_3 \sqrt{R^2 + X_3^2} / ^2 \\ U_5 &= I_5 Z_5 = I_5 \sqrt{R^2 + X_5^2} / ^2 \end{aligned} \right\} +$$

$$U_1^2 + U_3^2 + U_5^2 = I_1^2 (R^2 + X_1^2) + I_3^2 \left(R^2 + \frac{X_1^2}{9}\right) + I_5^2 \left(R^2 + \frac{X_1^2}{25}\right)$$

$$120^2 = (I_1^2 + I_3^2 + I_5^2) R^2 + \left(I_1^2 + \frac{I_3^2}{9} + \frac{I_5^2}{25}\right) X_1^2$$

$$X_1 = \sqrt{\frac{120^2 - (I_1^2 + I_3^2 + I_5^2) R^2}{I_1^2 + \frac{I_3^2}{9} + \frac{I_5^2}{25}}} = \sqrt{\frac{120^2 - 17,5 \cdot 100}{\frac{25+1+\frac{1}{25}}{2}}} = 31,17 \Omega = \frac{1}{\omega C}$$

$$C = \frac{1}{3140 \cdot 31,17} = 10,217 \mu F$$

10.2. Serijski RL spoj priključen je na napon $e = 100 + 100 \sin 1000t$ V. Treba odrediti efektivne vrijednosti napona i struje ako je $R = 30 \Omega$, $L = 40$ mH.

$$E = \sqrt{100^2 + \left(\frac{100}{\sqrt{2}}\right)^2} = 122,47 V$$

$$X_1 = \omega L = 10^3 \cdot 40 \cdot 10^{-3} = 40 \Omega$$

$$Z_1 = \sqrt{30^2 + 40^2} = 50 \Omega$$

$$\left. \begin{aligned} I_0 &= \frac{E_0}{R} = \frac{100}{30} A \\ I_1 &= \frac{E_1}{Z_1} = \frac{100}{\sqrt{2} \cdot 50} = \sqrt{2} \end{aligned} \right\} I = \sqrt{\left(\frac{100}{30}\right)^2 + 2} = 3,621 A$$

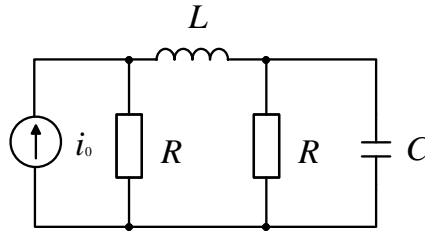
10.3. U zadatku 10.2. treba odrediti radnu i jalovu snagu izvora.

$$P = I^2 R = 3,621^2 \cdot 30 = 393,3 \text{ W}$$

$$Q = I_1^2 \cdot X_1 = 2 \cdot 40 = 80 \text{ VAR}$$

10.4. Treba odrediti struju i_L u shemi na slici, ako je $i_0 = 10 + 30\sqrt{2} \sin \omega t + 15\sqrt{2} \sin 2\omega t \text{ A}$,

$$R = \frac{1}{\omega C} = 1 \Omega, \quad \omega L = 0,5 \Omega.$$



a) $n = 0$: $I_{L0} = \frac{10}{2} = 5 \text{ A}$

b) $n = 1$: $\dot{Y}_{p1} = \frac{1}{R} + \frac{j}{X_C} = 1 + j = \sqrt{2} \angle 45^\circ \text{ S}$

$$\dot{Z}_{p1} = \frac{\sqrt{2}}{2} \angle -45^\circ = 0,5 - j0,5 \Omega$$

$$\dot{Z}'_1 = jX_L + \dot{Z}_{p1} = j0,5 + 0,5 - j0,5 = 0,5 \Omega$$

$$\dot{Z}_1 = \frac{1 \cdot 0,5}{1 + 0,5} = \frac{0,5}{1,5} = \frac{1}{3} \Omega \quad \Rightarrow \quad \dot{U}_1 = 30 \cdot \frac{1}{3} = 10 \text{ V} \quad \dot{I}_{L1} = \frac{\dot{U}_1}{\dot{Z}'_1} = \frac{10}{0,5} = 20 \text{ A}$$

c) $n = 2$: $X_{L2} = 1 \Omega$ $\dot{Y}_{p2} = \frac{1}{R} + \frac{j}{X_{C2}} = 1 + j2 = 2,236 \angle 63,435^\circ \text{ S}$

$$X_{C2} = 0,5 \Omega \quad \dot{Z}_{p2} = 0,44721 \angle -63,435^\circ = 0,2 - j0,4 \Omega$$

$$\dot{Z}'_2 = jX_{L2} + \dot{Z}_{p2} = j1 + 0,2 - j0,4 = 0,2 + j0,6 = 0,6324 \angle 71,565^\circ \Omega$$

$$\dot{Z}_2 = \frac{R \cdot \dot{Z}'_2}{R + \dot{Z}'_2} = \frac{1 \cdot 0,6324 \angle 71,565^\circ}{1 + 0,2 + j0,6} = \frac{0,6324 \angle 71,565^\circ}{1,34164 \angle 26,565^\circ} = 0,4714 \angle 45^\circ \Omega$$

$$\dot{U}_2 = 15 \cdot 0,4714 \angle 45^\circ = 7,071 \angle 45^\circ \text{ V}$$

$$\dot{I}_{L2} = \frac{\dot{U}_2}{\dot{Z}'_2} = \frac{7,071 \angle 45^\circ}{0,6324 \angle 71,565^\circ} = 11,18 \angle -26,565^\circ \text{ A}$$

ukupno: $i_L = 5 + 20\sqrt{2} \cdot \sin \omega t + 11,18\sqrt{2} \cdot \sin(2\omega t - 26,565^\circ) \text{ A}$

10.5. U zadatku 10.4. treba odrediti efektivnu vrijednost struje i_0 i radnu snagu izvora.

$$I_0 = \sqrt{10^2 + 30^2 + 15^2} = 35 \text{ A}$$

$$\text{a)} \quad P_0 = 10^2 \cdot \frac{1}{2} = 50 \text{ W}$$

$$\text{b)} \quad P_1' = \frac{U_1^2}{R} = \frac{100}{1} = 100 \text{ W}$$

$$\dot{U}_{p1} = \dot{I}_{L1} \cdot \dot{Z}_{p1} = 20 \cdot \frac{\sqrt{2}}{2} \angle -45^\circ \text{ W}$$

$$P_1'' = \frac{U_{p1}^2}{R} = \frac{100 \cdot 2}{1} = 200 \text{ W} \quad \Rightarrow \quad P_1 = 300 \text{ W}$$

$$\text{c)} \quad P_2' = \frac{U_2^2}{R} = \frac{7,071^2}{1} = 50 \text{ W}$$

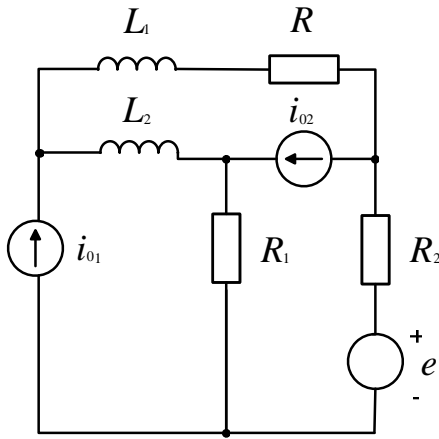
$$\dot{U}_{p2} = \dot{I}_{L2} \cdot \dot{Z}_{p2} = 11,18 \angle -26,565^\circ \cdot 0,44721 \angle -63,435^\circ = 5 \angle -90^\circ = -j5 \text{ V}$$

$$P_1'' = \frac{U_{p2}^2}{R} = \frac{25}{1} = 25 \text{ W} \quad \Rightarrow \quad P_2 = 75 \text{ W}$$

ukupno:

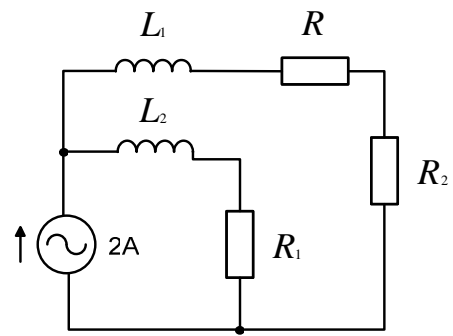
$$P = P_0 + P_1 + P_2 = 50 + 300 + 75 = 425 \text{ W}$$

10.6. U shemi na slici treba odrediti snagu na otporu $R = 5 \Omega$. Zadano: $i_{01} = 2\sqrt{2} \sin 100t \text{ A}$, $i_{02} = 10\sqrt{2} \sin (200t + 53^\circ) \text{ A}$, $e = 60 \sin 150t \text{ V}$, $L_1 = L_2 = 0,1 \text{ H}$, $R_1 = 15 \Omega$, $R_2 = 10 \Omega$.



superpozicija

a)

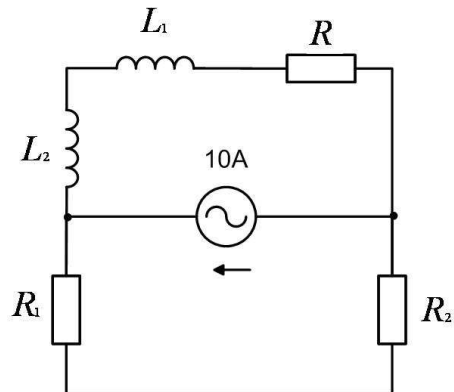


$$X_L' = \omega L = 100 \cdot 0,1 = 10 \Omega$$

$$\dot{I}_{R'} = 1 \text{ A}$$

$$P_R' = \dot{I}_{R'}^2 \cdot R = 1^2 \cdot 5 = 5 \text{ W}$$

b)



$$X_L'' = 200 \cdot 0,1 = 20 \, \Omega$$

$$\dot{Y}_1 = \frac{1}{5 + j40} = 0,003077 - j0,024615 = 0,024807 \angle -82,875^\circ \, S$$

$$\dot{Y}_2 = \frac{1}{25} = 0,04 \, S$$

$$\dot{Y} = \dot{Y}_1 + \dot{Y}_2 = 0,043077 - j0,024615 = 0,04961 \angle -29,745^\circ \, S$$

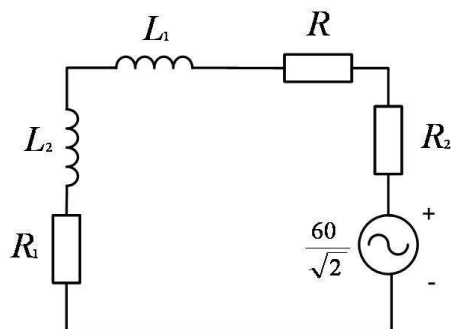
$$\dot{Z} = 20,1556 \angle 29,745^\circ \, \Omega$$

$$\dot{U}_p = 10 \angle 53^\circ \cdot 20,1556 \angle 29,745^\circ = 201,556 \angle 82,745^\circ \, V$$

$$\dot{I}''_R = \dot{U}_p \cdot \dot{Y}_1 = 201,556 \angle 82,745^\circ \cdot 0,024807 \angle -82,875^\circ = 5 \angle -0,13^\circ \, A$$

$$P_R'' = \dot{I}''_R{}^2 \cdot R = 5^2 \cdot 5 = 125 \, W$$

c)



$$X_L''' = 150 \cdot 0,1 = 15 \, \Omega$$

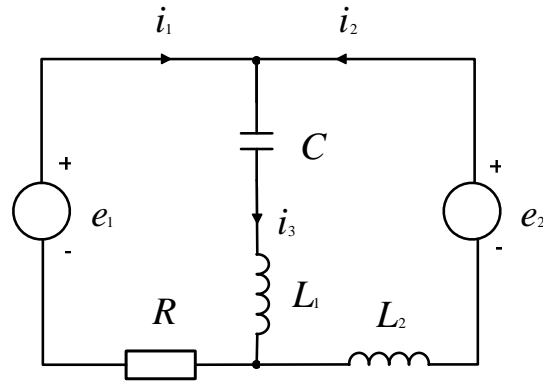
$$P_R''' = I'''^2 \cdot R = 1 \cdot 5 = 5 \, W$$

$$I''' = \frac{E}{Z} = \frac{60}{\sqrt{2} \cdot \sqrt{30^2 + 30^2}} = 1 \, A$$

ukupno:

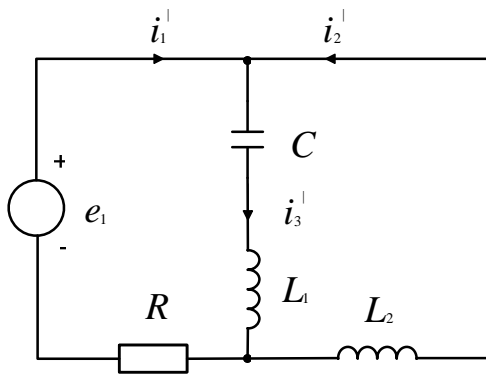
$$P = P_R' + P_R'' + P_R''' = 5 + 125 + 5 = 135 \, W$$

10.7. U shemi na slici treba odrediti trenutne struje u svim granama, radne snage na otporu R i izvoru e_1 . Zadano: $e_1 = 6 + 3 \sin(\omega t + 30^\circ) + 6 \sin 2\omega t$ V, $e_2 = 3$ V, $R = 6 \Omega$, $\omega L_1 = 1 \Omega$, $\omega L_2 = 3 \Omega$, $1/\omega C = 4 \Omega$.



superpozicija

a) prvi izvor



$$1^\circ \quad E_{10} = 6 \text{ V}; \quad I_{30a} = 0$$

$$I_{10a} = -I_{20a} = \frac{E_{10}}{R} = \frac{6}{6} = 1 \text{ A}$$

$$2^\circ \omega: \quad X_{C1} = \frac{1}{\omega C} = 4 \Omega,$$

$$X_{L11} = 1 \Omega, \quad X_{L21} = 3 \Omega$$

$$\dot{E}_{11} = \frac{3}{\sqrt{2}} \angle 30^\circ \text{ V}$$

$$\dot{Y}_1 = \frac{1}{j(X_{L11} - X_{C1})} = -\frac{j}{1-4} = \frac{j}{3}$$

$$\dot{Y}_2 = -\frac{j}{3}$$

$$\dot{Y}_p = \dot{Y}_1 + \dot{Y}_2 = \frac{j}{3} - \frac{j}{3} = 0; \quad \dot{Z}_p = \infty \Rightarrow \dot{I}_{11} = 0$$

$$\dot{I}_{21} = -\dot{E}_{11} \cdot \dot{Y}_2 = -\frac{3}{\sqrt{2}} \angle 30^\circ \cdot \left(-\frac{1 \angle 90^\circ}{3} \right) = \frac{1}{\sqrt{2}} \angle 120^\circ \text{ A} = -\frac{1}{\sqrt{2}} \angle -60^\circ \text{ A}$$

$$\dot{I}_{31} = \dot{E}_{11} \cdot \dot{Y}_1 = \frac{3 \angle 30^\circ}{\sqrt{2}} \cdot \frac{1 \angle 90^\circ}{3} = \frac{1}{\sqrt{2}} \angle 120^\circ \text{ A}$$

$$3^\circ 2\omega: \quad X_{C2} = 2 \Omega$$

$$X_{L12} = 2 \Omega$$

$$X_{L22} = 6 \Omega$$

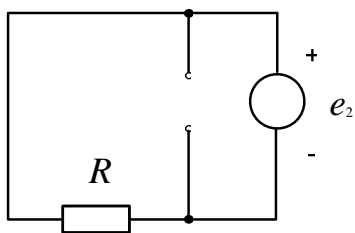
$$\dot{E}_{12} = \frac{6}{\sqrt{2}} \angle 0^\circ \text{ V}$$

$$\dot{Y}_1 = \frac{1}{j(X_{L12} - X_{C2})} = \frac{-j}{2-2} = \infty \quad \Rightarrow \quad \dot{Z}_1 = 0$$

$$\dot{I}_{12} = \dot{I}_{32} = \frac{\dot{E}_{12}}{R} = \frac{6}{\sqrt{2} \cdot 6} = \frac{1}{\sqrt{2}} \angle 0^\circ \text{ A}$$

$$\dot{I}_{22} = 0$$

b) drugi izvor



$$I_{10b} = -I_{20b} = \frac{-E_2}{R} = -\frac{3}{6} = -0,5 \text{ A}$$

$$I_{30b} = 0$$

ukupno:

$$i_1 = 0,5 + \sin 2\omega t \quad \Rightarrow \quad I_1 = \sqrt{0,5^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 0,866 \text{ A}$$

$$i_2 = -0,5 - \sin(\omega t - 60^\circ) \quad \Rightarrow \quad I_2 = \sqrt{0,5^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 0,886 \text{ A}$$

$$i_3 = \sin(\omega t + 120^\circ) + \sin 2\omega t \quad \Rightarrow \quad I_3 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1 \text{ A}$$

$$P_R = I_1^2 \cdot R = 0,866^2 \cdot 6 = 4,5 \text{ W}$$

$$P_{e'10} = 6 \cdot 0,5 = 3 \text{ W}$$

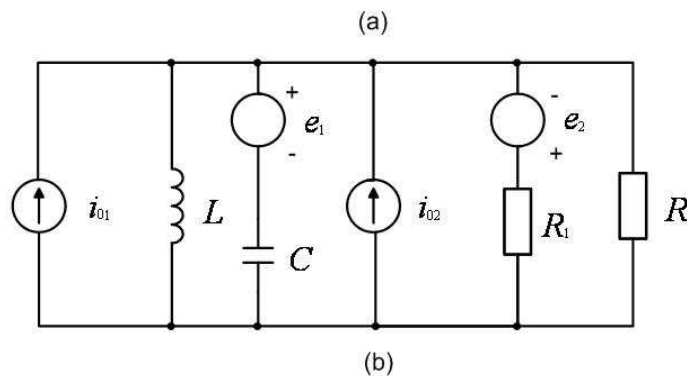
$$P_{e12}' = \operatorname{Re} \left\{ \frac{6}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right\} = \frac{6}{2} = 3 \text{ W} \quad \Rightarrow \quad P_{e1} = 6 \text{ W}$$

10.8. U mreži prema slici treba odrediti napon u_{ab} i radnu snagu na otporu $R = 4 \Omega$.

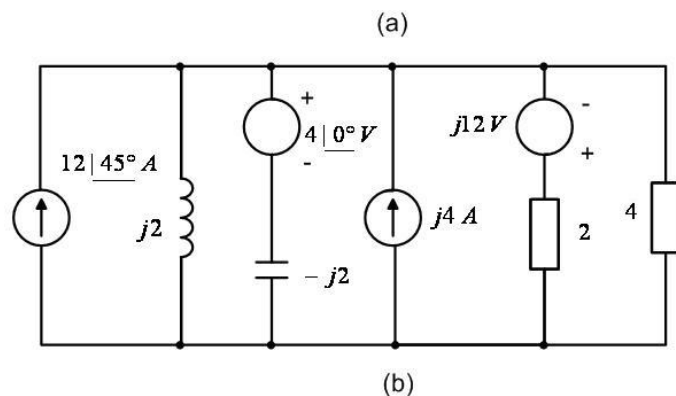
Zadano: $\omega L = 1/\omega C = R_1 = 2 \Omega$, $e_1 = 4\sqrt{2} \sin \omega t$ V,

$e_2 = 12\sqrt{2} \sin(\omega t + 90^\circ)$ V, $i_{01} = 12\sqrt{2} \sin(\omega t + 45^\circ) - 12 \sin(2\omega t - 45^\circ)$ A,

$i_{02} = 4\sqrt{2} \sin(\omega t + 90^\circ)$ A.

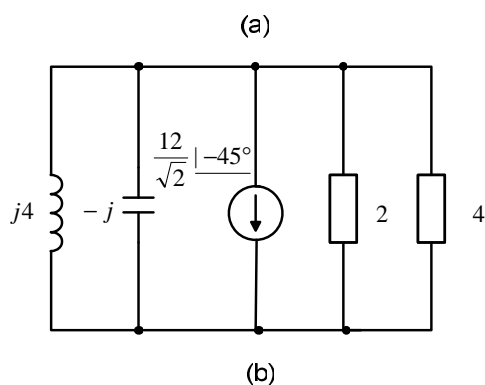


a) $n = 1$



$$\begin{aligned} \dot{U}_{ab1} &= \frac{12 \angle 45^\circ + 4 \cdot 0,5j + j4 - j12 \cdot 0,5}{-j0,5 + j0,5 + 0,5 + 0,25} \\ &= \frac{12 \angle 45^\circ}{0,75} = 16 \angle 45^\circ \text{ V} \end{aligned}$$

b) $n = 2$

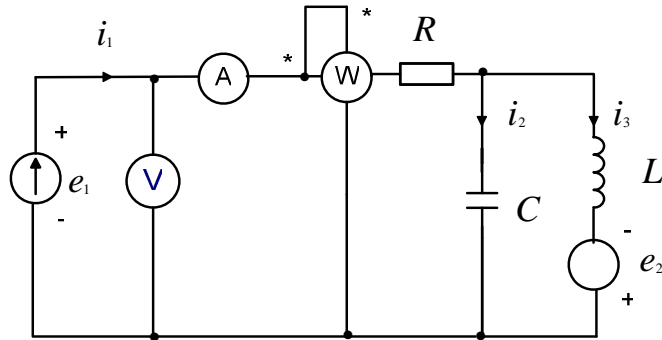


$$\begin{aligned} \dot{U}_{ab2} &= \frac{-6\sqrt{2} \angle -45^\circ}{-j0,25 + j + 0,5 + 0,25} = \frac{6\sqrt{2} \angle 135^\circ}{0,75 + j0,75} \\ &= \frac{6\sqrt{2} \angle 135^\circ}{0,75\sqrt{2} \angle 45^\circ} = 8 \angle 90^\circ = -8 \angle -90^\circ \text{ V} \end{aligned}$$

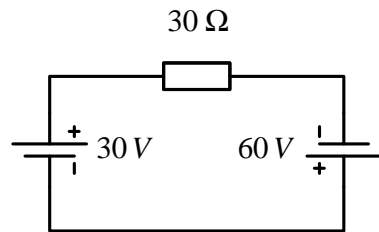
$$U_{ab} = \sqrt{U_{ab1}^2 + U_{ab2}^2} = \sqrt{16^2 + 8^2} = \sqrt{320} \Rightarrow P_R = \frac{U_{ab}^2}{R} = \frac{320}{4} = 80 \text{ W}$$

$$u_{ab} = 16\sqrt{2} \sin(\omega t + 45^\circ) - 8\sqrt{2} \sin(2\omega t - 90^\circ) \text{ V}$$

10.9. U mreži na slici treba odrediti trenutne struje grana i vrijednosti koje pokazuju idealni instrumenti. Zadano: $R = 30 \Omega$, $\omega L = 30 \Omega$, $1/\omega C = 120 \Omega$,
 $e_1 = 30 + 141 \sin(\omega t + 30^\circ) + 85 \sin(2\omega t - 15^\circ) \text{ V}$, $e_2 = 60 \text{ V}$.

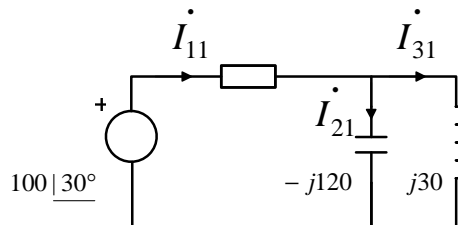


a) $n = 0$:



$$I_{10} = \frac{30 + 60}{30} = \frac{90}{30} = 3 \text{ A} = I_{30}$$

b) $n = 1$:



$$\dot{Y}_p = j\frac{1}{120} - j\frac{1}{30} = j\frac{1-4}{120} = -j\frac{3}{120} = \frac{1}{j40} \text{ S}$$

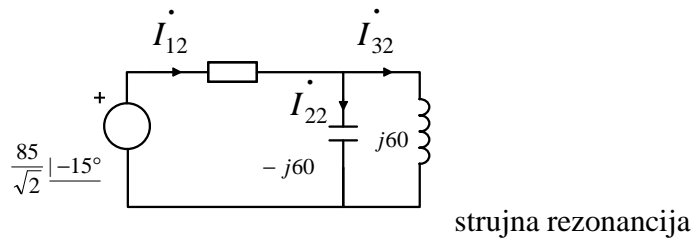
$$\dot{Z}_p = j40 \Omega \Rightarrow \dot{Z} = 30 + j40 = 50 \angle 53,13^\circ \Omega$$

$$\dot{U}_p = \dot{I}_{11} \cdot \dot{Z}_p = 2 \angle -23,13^\circ - 40 \angle 90^\circ = 80 \angle 66,87^\circ \text{ V}$$

$$\dot{I}_{21} = \frac{\dot{U}_p}{-j120} = \frac{80 \angle 66,87^\circ}{120 \angle -90^\circ} = 0,6 \angle 156,87^\circ = \frac{0,94281}{\sqrt{2}} \angle 156,87^\circ \text{ A}$$

$$\dot{I}_{31} = \frac{\dot{U}_p}{j30} = \frac{80 \angle 66,87^\circ}{30 \angle 90^\circ} = 2,6 \angle -23,13^\circ = \frac{3,771}{\sqrt{2}} \angle -23,13^\circ \text{ A}$$

c) $n = 2$:



$$\dot{I}_{12} = 0 \quad \Rightarrow \quad \dot{I}_{22} = \frac{\dot{U}_p}{-j60} = \frac{85 \angle -15^\circ}{\sqrt{2} \cdot 60 \angle -90^\circ} = 1,0017 \angle 75^\circ \text{ A}$$

$$\dot{I}_{32} = \frac{\dot{U}_p}{j60} = \frac{85 \angle -15^\circ}{\sqrt{2} \cdot 60 \angle 90^\circ} = 1,0017 \angle -105^\circ \text{ A}$$

$$i_1 = 3 + 2,828 \cdot \sin(\omega t - 23,13^\circ) \text{ A} \quad \Rightarrow \quad I_A = \sqrt{3^2 + 2^2} = \sqrt{13} = 3,606 \text{ A}$$

$$i_2 = 0,94281 \cdot \sin(\omega t + 156,87^\circ) + \sqrt{2} \cdot \sin(2\omega t + 75^\circ) \text{ A}$$

$$i_3 = 3 + 3,771 \cdot \sin(\omega t - 23,13^\circ) + \sqrt{2} \cdot \sin(2\omega t - 105^\circ) \text{ A}$$

$$U_V = \sqrt{30^2 + \left(\frac{141}{\sqrt{2}}\right)^2 + \left(\frac{85}{\sqrt{2}}\right)^2} = 120,22 \text{ V}$$

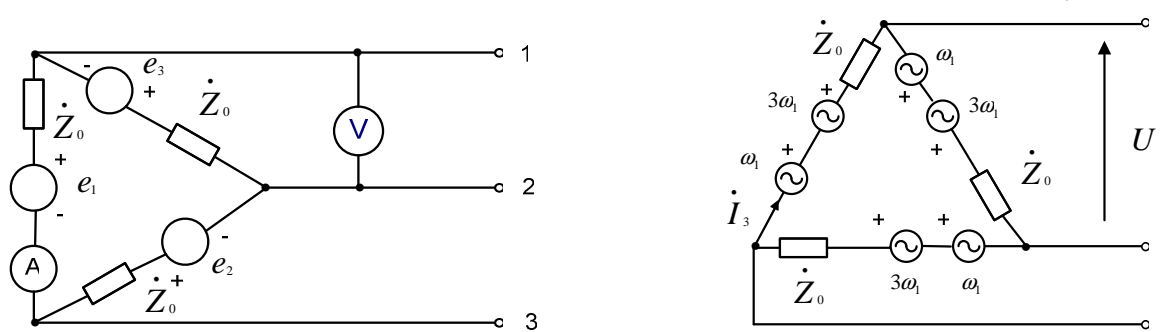
$$P_0 = I_{10}^2 \cdot R = 3^2 \cdot 30 = 270 \text{ W}$$

$$P_1 = \operatorname{Re}\{\dot{U}_1 \cdot \dot{I}_{11}^*\} = \operatorname{Re}\{100 \angle 30^\circ \cdot 2 \angle 23,13^\circ\} = \operatorname{Re}\{200 \angle 53,13^\circ\} = \operatorname{Re}\{120 + j160\} = 120 \text{ W}$$

$$P = P_0 + P_1 = 270 + 120 = 390 \text{ W} = I_A^2 \cdot R = 13 \cdot 30$$

10.10. U shemi na slici treba odrediti što pokazuju instrumenti, ako je jedna faza simetričnog

generatora $e_1 = 141 \sin \omega t + 10,6 \sin 3\omega t$, a unutrašnja impedancija $\dot{Z}_0 = j\omega L_0 = j1 \Omega$

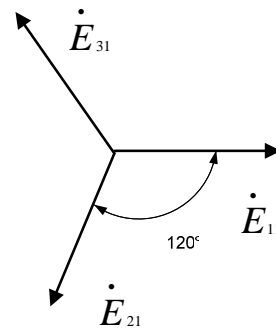
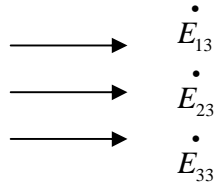


Fazori trećih harmonika sve tri faze imaju kut nula (u fazi su). Stoga tjeraju kružnu struju trećeg harmonika u namotima generatora koja stvara padove napona na unutrašnjim impedancijama svih triju faza. Ti padovi napona su suprotnog predznaka od EMS trećeg harmonika u svakoj fazi. Kad je generator simetričan, napon trećeg harmonika se ne može izmjeriti u linijskom naponu.

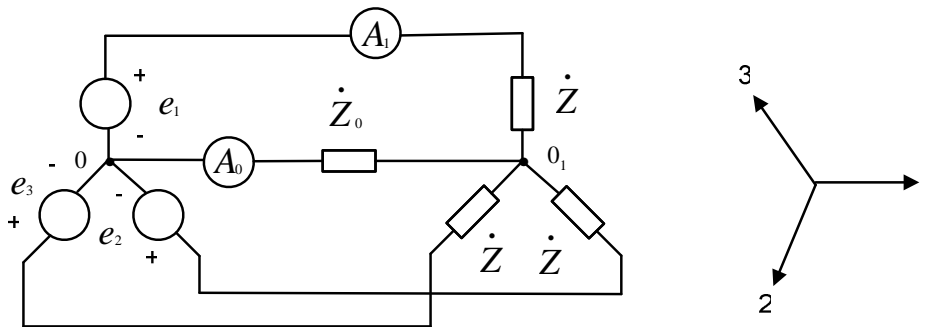
$$U_V = U_1 = 100 \text{ V}$$

$$\dot{I}_3 = \frac{\dot{E}_3}{\dot{Z}_{03}} = \frac{10,6 \angle 0^\circ}{\sqrt{2} \cdot j3} = 2,495 \angle -90^\circ \text{ A}$$

$$I_A = 2,5 \text{ A}$$



10.11. U shemi na slici zadana je jedna faza simetričnog generatora $e = 100 \sin \omega t + 30 \sin 3\omega t \text{ V}$, impedancija simetričnog trošila $\dot{Z}(n) = R + jn\omega L = 1 + j3n \Omega$, te impedancija nulvodiča $\dot{Z}_0 = R_0 + jn\omega L_0 = 2 + jn \Omega$, gdje je n - broj harmonika. Treba odrediti što pokazuju instrumenti.



a) $n = 1$: $\dot{E}_{11} = \frac{100}{\sqrt{2}} \angle 0^\circ \text{ V}$ $\dot{Z}_1 = 1 + j3 = 3,162 \angle 71,565^\circ \Omega$

$$\dot{U}'_{0,0} = U \cdot \frac{1 \angle 0^\circ \cdot \frac{1}{\dot{Z}_1} + 1 \angle -120^\circ \cdot \frac{1}{\dot{Z}_1} + 1 \angle 120^\circ \cdot \frac{1}{\dot{Z}_1}}{\frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_1} + \frac{1}{\dot{Z}_{01}}}$$

$$\dot{U}'_{0,0} = \frac{U}{\dot{Z}_1} \cdot \frac{0}{\frac{3}{\dot{Z}_1} + \frac{1}{\dot{Z}_{01}}} = 0$$

$$\dot{I}_1 = \frac{\dot{E}_1}{\dot{Z}_1} = \frac{100}{\sqrt{2} \cdot 3,162 \angle 71,565^\circ} = 22,36 \angle -71,565^\circ \text{ A}$$

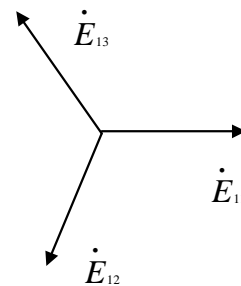
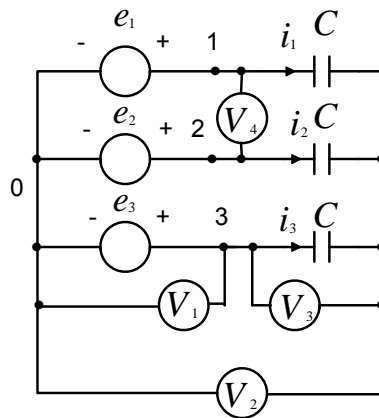
b) $n = 3$: $\dot{E}_3 = \frac{30}{\sqrt{2}} \angle 0^\circ \text{ V}$ $\dot{Z}_3 = 1 + j9 \Omega$ $\dot{Z}_{03} = 2 + j3 \Omega$

$$\begin{aligned}
 \dot{U}'''_{0,0} &= \dot{E}_3 \cdot \frac{\frac{1}{\dot{Z}_3} + \frac{1}{\dot{Z}_3} + \frac{1}{\dot{Z}_3}}{\frac{1}{\dot{Z}_3} + \frac{1}{\dot{Z}_3} + \frac{1}{\dot{Z}_3} + \frac{1}{\dot{Z}_{03}}} = \frac{\frac{3 \cdot \dot{E}_3}{\dot{Z}_3}}{\frac{3}{\dot{Z}_3} + \frac{1}{\dot{Z}_{03}}} = \frac{\dot{Z}_3}{\frac{3}{\dot{Z}_3} + \frac{1}{\dot{Z}_{03}}} \cdot \frac{\dot{Z}_3}{3} \\
 \dot{U}'''_{0,0} &= \frac{\dot{E}_3}{1 + \frac{\dot{Z}_3}{3 \cdot \dot{Z}_{03}}} = \frac{\dot{E}_3}{\frac{3 \cdot \dot{Z}_{03} + \dot{Z}_3}{3 \cdot \dot{Z}_{03}}} = \frac{3 \cdot \dot{E}_3 \cdot \dot{Z}_{03}}{3 \cdot \dot{Z}_{03} + \dot{Z}_3} \\
 \dot{I}_{03} &= \frac{\dot{U}'''_{0,0}}{\dot{Z}_{03}} = \frac{3 \cdot \dot{E}_3 \cdot \dot{Z}_{03}}{3 \cdot \dot{Z}_{03} + \dot{Z}_3} \cdot \frac{1}{\dot{Z}_{03}} = \frac{3 \cdot \dot{E}_3}{3 \cdot \dot{Z}_{03} + \dot{Z}_3} = \frac{3 \cdot \dot{E}_3}{6 + j9 + 1 + j9} = \frac{3 \cdot \dot{E}_3}{7 + j18} \\
 &= \frac{90 \angle 0^\circ}{\sqrt{2} \cdot 19,3132 \angle 68,74949^\circ} = 3,2951 \angle -68,75^\circ \text{ A} \quad \Rightarrow \quad I_{A0} = 3,295 \text{ A}
 \end{aligned}$$

Ukupno:
$$I_{A1} = \sqrt{22,36^2 + \left(\frac{3,295}{3}\right)^2} = 22,39 \text{ A}.$$

10.12. Na simetrični generator, prema slici, čija je EMS jedne faze

$e = 120 \sin \omega t + 20 \sin (3\omega t + 30^\circ) + 12 \sin (5\omega t - 30^\circ) \text{ V}$, priključeni su u zvijezdu jednaki kondenzatori $1/\omega C = 30 \Omega$. Treba odrediti struje i_1 , i_3 te pokazivanje voltmetara.

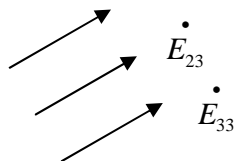


a) $n = 1$:

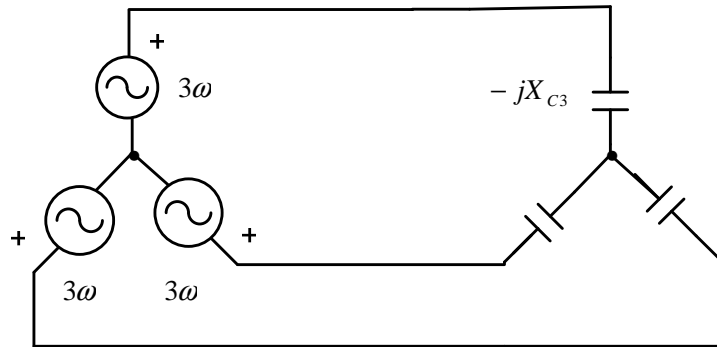
$$\dot{E}_{11} = \frac{120}{\sqrt{2}} \angle 0^\circ \text{ V} \quad \Rightarrow \quad \dot{I}_{11} = \frac{\dot{E}_{11}}{-jX_{C1}} = \frac{120 \angle 0^\circ}{\sqrt{2} \cdot 30 \angle -90^\circ} = 2\sqrt{2} \angle 90^\circ \text{ A}$$

b) $n = 3$:

$$\dot{E}_{13} = \dot{E}_3 = \frac{20}{\sqrt{2}} \angle 30^\circ \text{ V}$$



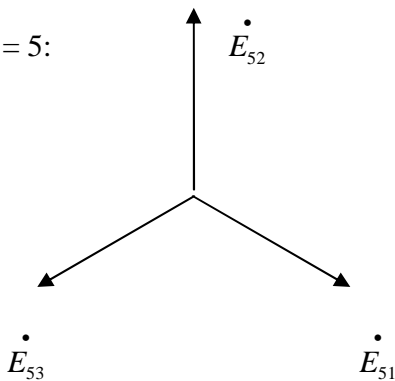
$$30^\circ + 3 \cdot 120^\circ = 30^\circ + 360^\circ = 30^\circ$$



$$\dot{U}_{0,0} = \frac{\frac{\dot{E}_3}{-jX_C} + \frac{\dot{E}_3}{-jX_C} + \frac{\dot{E}_3}{-jX_C}}{-\frac{3}{jX_C}} = \frac{-\frac{3 \cdot \dot{E}_3}{jX_C}}{-\frac{3}{jX_C}} = \dot{E}_3$$

$$\dot{I}_{31} = \frac{\dot{E}_3 - \dot{U}_{0,0}}{-jX_{C3}} = \frac{0}{-jX_{C3}} = 0 \text{ A}$$

c) $n = 5$:



$$\dot{E}_{51} = \frac{12}{\sqrt{2}} \angle -30^\circ \text{ V}$$

$$5 \cdot 120^\circ = 600^\circ = 240^\circ = -120^\circ$$

$$5 \cdot (-120^\circ) = -600^\circ = -240^\circ = 120^\circ$$

$$\dot{I}_{51} = \frac{\dot{E}_5}{-jX_{C5}} = \frac{12 \angle -30^\circ}{\sqrt{2} \cdot 6 \angle -90^\circ} = \sqrt{2} \angle 60^\circ \text{ A}$$

Ukupno:

$$i_1 = 4 \cdot \sin(\omega t + 90^\circ) + 2 \sin(5\omega t + 60^\circ) \text{ A}$$

$$i_2 = 4 \cdot \sin(\omega t - 30^\circ) + 2 \sin(5\omega t + 180^\circ) \text{ A}$$

$$i_3 = 4 \cdot \sin(\omega t + 210^\circ) + 2 \sin(5\omega t - 60^\circ) \text{ A}$$

$$U_{V1} = \sqrt{\frac{120^2 + 20^2 + 12^2}{2}} = 86,44 \text{ V}$$

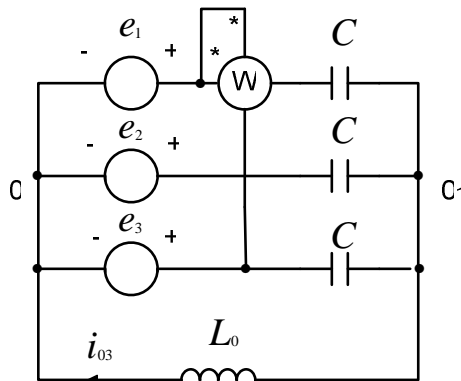
$$U_{V2} = E_3 = \frac{20}{\sqrt{2}} = 14,14 \text{ V}$$

$$U_{V3} = \sqrt{E_1^2 + E_5^2} = \sqrt{\frac{120^2 + 12^2}{2}} = 85,276 \text{ V}$$

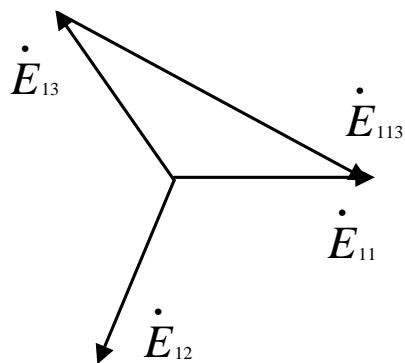
$$U_{V4} = \sqrt{E_{11}^2 + E_{51}^2} = \sqrt{\left(\frac{120}{\sqrt{2}} \cdot \sqrt{3}\right)^2 + \left(\frac{12}{\sqrt{2}} \cdot \sqrt{3}\right)^2} = \sqrt{(120^2 + 12^2) \cdot 1,5} = 147,7 \text{ V}.$$

10.13. Ako se u shemi na slici između zvjezdišta spoji nulvodič impedancije $\dot{Z}_0 = j\omega L_0 = j1\Omega$, treba odrediti: a) struje i_L , i_3 , i_0 ; b) pokazivanje vatmetra čiji je strujni svitak spojen u fazu (1), a naponski svitak je spojen između faza (1) i (3).

Zadano: $\frac{1}{\omega C} = 30\Omega$, $e = 120 \sin \omega t + 20 \sin (3\omega t + 30^\circ) + 12 \sin (5\omega t - 30^\circ) \text{ V}$.



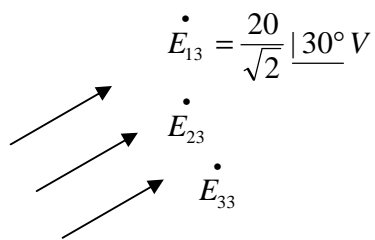
a) $n = 1$:



$$\dot{E}_{11} = \frac{120}{\sqrt{2}} \angle 0^\circ \text{ V}$$

$$\dot{I}_{11} = \frac{\dot{E}_{11}}{-jX_{C1}} = \frac{120 \angle 0^\circ}{\sqrt{2} \cdot 30 \angle -90^\circ} = 2\sqrt{2} \angle 90^\circ \text{ A}$$

b) $n = 3$:

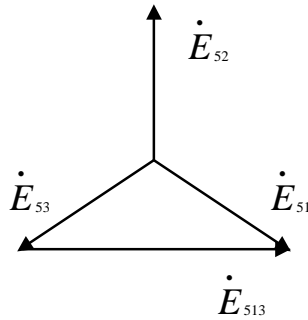


$$30^\circ + 3 \cdot 120^\circ = 30^\circ + 360^\circ = 30^\circ$$

vidi zadatak 10.11.:

$$\dot{I}_{03} = \frac{3 \cdot \dot{E}_{13}}{3 \cdot \dot{Z}_{03} + \dot{Z}} = \frac{3 \cdot \frac{20}{\sqrt{2}} \angle 30^\circ}{3 \cdot j3 - j10} = \frac{\frac{60}{\sqrt{2}} \angle 30^\circ}{1 \cdot \angle -90^\circ} = \frac{60}{\sqrt{2}} \angle 120^\circ \text{ A} \Rightarrow i_0 = 60 \cdot \sin(3\omega t + 120^\circ) \text{ A}$$

c) $n = 5$:



$$5 \cdot 120^\circ = 600^\circ = 240^\circ = -120^\circ$$

$$5 \cdot (-120^\circ) = -600^\circ = -240^\circ = 120^\circ$$

$$\dot{E}_{s1} = \frac{12}{\sqrt{2}} \angle -30^\circ \text{ V}$$

$$\dot{I}_{s1} = \frac{\dot{E}_s}{-jX_{C5}} = \frac{12 \angle -30^\circ}{\sqrt{2} \cdot 6 \angle -90^\circ} = \sqrt{2} \angle 60^\circ \text{ A}$$

Ukupno:

$$i_1 = 4 \cdot \sin(\omega t + 90^\circ) + 20 \cdot \sin(3\omega t + 120^\circ) + 2 \sin(5\omega t + 60^\circ) \text{ A}$$

$$i_2 = 4 \cdot \sin(\omega t - 30^\circ) + 20 \cdot \sin(3\omega t + 120^\circ) + 2 \sin(5\omega t + 180^\circ) \text{ A}$$

$$i_3 = 4 \cdot \sin(\omega t + 210^\circ) + 20 \cdot \sin(3\omega t + 120^\circ) + 2 \sin(5\omega t - 60^\circ) \text{ A}$$

$$p(t) = i_1 \cdot e_{13}$$

$$P_1 = E_{113} \cdot I_{11} \cdot \cos \varphi_1 = \frac{120}{\sqrt{2}} \cdot \sqrt{3} \cdot 2\sqrt{2} \cdot \cos(-30^\circ - 90^\circ) = -207,84 \text{ W}$$

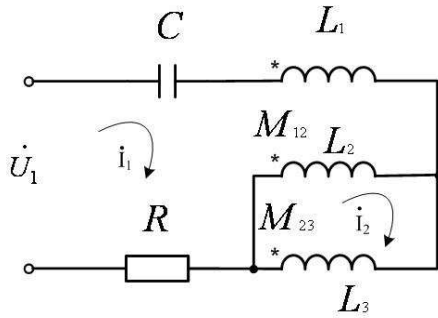
$$P_3 = E_{313} \cdot I_{13} \cdot \cos \varphi_3 = 0 \cdot \frac{20}{\sqrt{2}} \cdot \cos(30^\circ - 120^\circ) = 0 \text{ W}$$

$$P_5 = E_{513} \cdot I_{15} \cdot \cos \varphi_5 = \frac{12}{\sqrt{2}} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(0^\circ - 60^\circ) = 10,39 \text{ W}$$

$$P_W = P_1 + P_3 + P_5 = -207,846 + 0 + 10,392 = -197,45 \text{ W}$$

11. Linearne mreže izmjenične struje s međuinduktivnim vezama

11.1. Treba odrediti kapacitet C u shemi na slici ako je rezonantna frekvencija $f_0 = 16 \text{ kHz}$, a $L_1 = 2,8 \text{ mH}$, $L_2 = 6 \text{ mH}$, $L_3 = 3 \text{ mH}$ i $M_{12} = M_{23} = 2 \text{ mH}$.



$$\omega = 2 \cdot \pi \cdot f = 2 \cdot \pi \cdot 16 \cdot 10^3 = 100,53 \cdot 10^3 \text{ rad/s}$$

$$X_{L1} = \omega \cdot L_1 = 100,53 \cdot 10^3 \cdot 2,8 \cdot 10^{-3} = 281,49 \Omega$$

$$X_{L2} = \omega \cdot L_2 = 100,53 \cdot 10^3 \cdot 6 \cdot 10^{-3} = 603,19 \Omega$$

$$X_{L3} = \omega \cdot L_3 = 100,53 \cdot 10^3 \cdot 3 \cdot 10^{-3} = 301,59 \Omega$$

$$X_M = \omega \cdot M = 100,53 \cdot 10^3 \cdot 2 \cdot 10^{-3} = 201,06 \Omega$$

$$\dot{U}_1 = \dot{I}_1 \cdot (-jX_C + jX_{L1} - jX_M + R) + (\dot{I}_1 - \dot{I}_2) \cdot (jX_{L2} - jX_M) + \dot{I}_2 \cdot jX_M$$

$$0 = (\dot{I}_1 - \dot{I}_2) \cdot (jX_M - jX_{L2}) + \dot{I}_1 \cdot jX_M - \dot{I}_2 \cdot jX_M + \dot{I}_2 \cdot jX_{L3}$$

$$\dot{U}_1 = \dot{I}_1 \cdot [R + j \cdot (X_{L1} - X_M - X_C)] + \dot{I}_1 \cdot j \cdot (X_{L2} - X_M) + \dot{I}_2 \cdot j \cdot (X_M - X_{L2}) + \dot{I}_2 \cdot jX_M$$

$$0 = \dot{I}_1 \cdot j \cdot (X_M - X_{L2}) + \dot{I}_2 \cdot j \cdot (X_{L2} - X_M) + \dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot j \cdot (X_{L3} - X_M)$$

$$\dot{U}_1 = \dot{I}_1 \cdot [R + j \cdot (X_{L1} + X_{L2} - 2 \cdot X_M - X_C)] + \dot{I}_2 \cdot j \cdot (2 \cdot X_M - X_{L2})$$

$$0 = \dot{I}_1 \cdot j \cdot (2 \cdot X_M - X_{L2}) + \dot{I}_2 \cdot j \cdot (X_{L2} + X_{L3} - 2 \cdot X_M)$$

$$\dot{I}_2 = -\frac{2 \cdot X_M - X_{L2}}{X_{L2} + X_{L3} - 2 \cdot X_M} \cdot \dot{I}_1$$

$$\dot{U}_1 = \dot{I}_1 \cdot [R + j \cdot (X_{L1} + X_{L2} - 2 \cdot X_M - X_C)] - j \cdot \frac{2 \cdot X_M - X_{L2}}{X_{L2} + X_{L3} - 2 \cdot X_M} \cdot \dot{I}_1 \cdot (2 \cdot X_M - X_{L2})$$

$$\frac{\dot{U}_1}{\dot{I}_1} = R + j \cdot (281,49 + 603,19 - 2 \cdot 201,06 - X_C) - j \cdot \frac{(2 \cdot 201,06 - 603,19)^2}{603,19 + 301,59 - 2 \cdot 201,06} = \dot{Z}_u$$

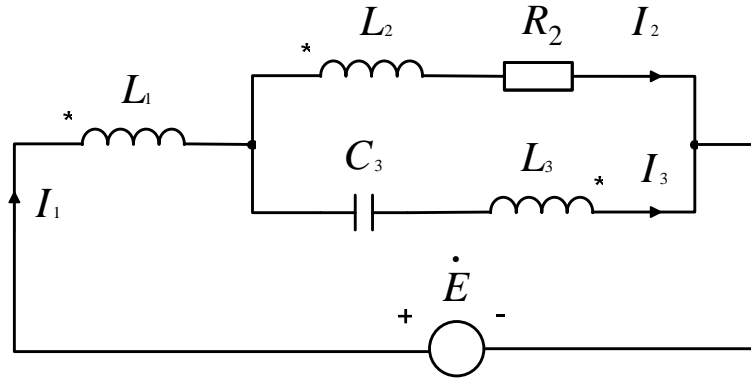
$$\text{Im}\left\{\dot{Z}_u\right\} = 0 \rightarrow 482,55 - X_C = \frac{(-201,06)^2}{502,65} = 80,42$$

$$X_C = 482,55 - 80,42 = 402,12 = \frac{1}{\omega C}$$

$$C = \frac{1}{100,53 \cdot 10^3 \cdot 402,12} = 24,737 \text{ nF}$$

11.2. U shemi na slici treba odrediti struje grana i nacrtati fazorski dijagram.

Zadano: $R_2 = X_{L3} = 2 \Omega$, $X_{L2} = X_{L1} = X_{C3} = 4 \Omega$, $\dot{E} = 10 \angle 0^\circ \text{ V}$, faktori međuinduktivnih veza iznose: $k_{12} = 0,5$, $k_{23} = 0,707$, $k_{31} = 0$.



$$X_{12} = k_{12} \cdot \sqrt{X_{L1} \cdot X_{L2}} = 0,5 \cdot \sqrt{4 \cdot 4} = 0,5 \cdot 4 = 2 \Omega$$

$$X_{23} = k_{23} \cdot \sqrt{X_{L2} \cdot X_{L3}} = 0,707 \cdot \sqrt{4 \cdot 2} = 2 \Omega$$

$$\dot{E} = \dot{I}_1 \cdot jX_{L1} + \dot{I}_2 \cdot jX_{L2} + \dot{I}_3 \cdot (jX_{L3} - jX_{C3}) - \dot{I}_2 \cdot jX_{23}$$

$$0 = \dot{I}_2 \cdot (R_2 + jX_{L2}) + \dot{I}_1 \cdot jX_{L2} - \dot{I}_3 \cdot jX_{23} - \dot{I}_3 \cdot (jX_{L3} - jX_{C3})$$

$$\dot{I}_1 = \dot{I}_2 + \dot{I}_3$$

$$\dot{E} = \dot{I}_1 \cdot j4 + \dot{I}_2 \cdot j \cdot (2 - 2) + \dot{I}_3 \cdot j \cdot (2 - 4)$$

$$0 = \dot{I}_1 \cdot j2 + \dot{I}_2 \cdot [2 + j \cdot (4 + 2)] - \dot{I}_3 \cdot j \cdot (2 + 2 - 4)$$

$$\dot{I}_1 = \dot{I}_2 + \dot{I}_3$$

$$10 = j4 \cdot \dot{I}_2 + j4 \cdot \dot{I}_3 - j2 \cdot \dot{I}_3 = j4 \cdot \dot{I}_2 + j2 \cdot \dot{I}_3$$

$$0 = j2 \cdot \dot{I}_2 + j2 \cdot \dot{I}_3 + \dot{I}_2 \cdot (2 + j6) = \dot{I}_2 \cdot (2 + j8) + j2 \cdot \dot{I}_3$$

$$\dot{I}_3 = \frac{2 + j8}{-j2} \cdot \dot{I}_2 = (-4 + j) \cdot \dot{I}_2$$

$$10 = j4 \cdot \dot{I}_2 + j2 \cdot (-4 + j) \cdot \dot{I}_2 = \dot{I}_2 \cdot (j4 - 2 - j8) = (-2 - j4) \cdot \dot{I}_2$$

$$\dot{I}_2 = \frac{10}{-2 - j4} = -1 + j2 = 2,236 \angle 116,565^\circ \text{ A}$$

$$\dot{I}_3 = (-4 + j) \cdot (-1 + j2) = 2 - j9 = 9,22 \angle -77,47^\circ \text{ A}$$

$$\underline{\dot{I}_1 = \dot{I}_2 + \dot{I}_3 = -1 + j2 + 2 - j9 = 1 - j7 = 7,701 \angle -81,87^\circ \text{ A}}$$

$$\dot{U}_{L1} = \dot{I}_1 \cdot jX_{L1} + \dot{I}_2 \cdot jX_{L2} = (1 - j7) \cdot j4 + (-1 + j2) \cdot j2 = j4 + 28 - j2 - 4 = 24 + j2 \text{ V}$$

$$\dot{U}_{C3} = \dot{I}_3 \cdot (-jX_{C3}) = (2 - j9) \cdot (-j4) = -j8 - 36 = -36 - j8 \text{ V}$$

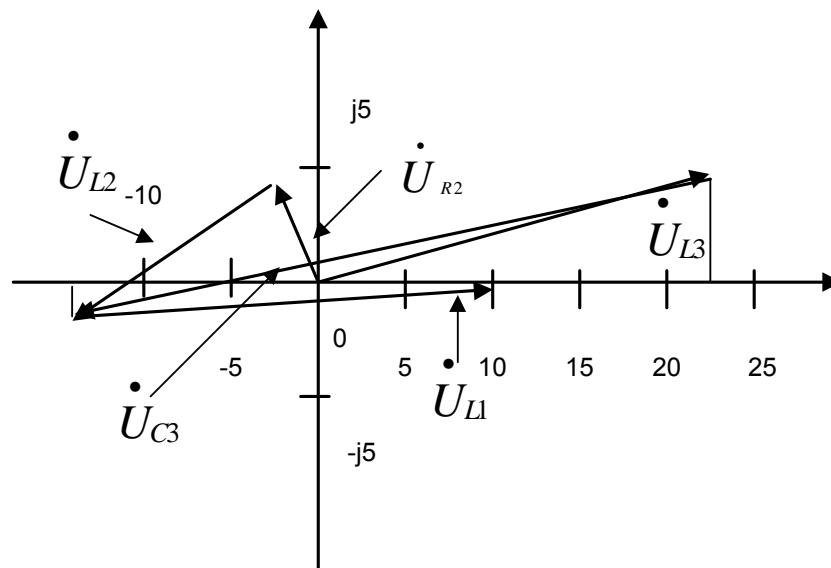
$$\dot{U}_{L3} = \dot{I}_3 \cdot jX_{L3} - \dot{I}_2 \cdot jX_{23} = (2 - j9) \cdot j2 - (-1 + j2) \cdot j2 = j4 + 18 + j2 + 4 = 22 + j6 \text{ V}$$

$$\underline{\dot{E} = \dot{U}_{L1} + \dot{U}_{C3} + \dot{U}_{L3} = 24 + j2 - 36 - j8 + 22 + j6 = 10 \text{ V}}$$

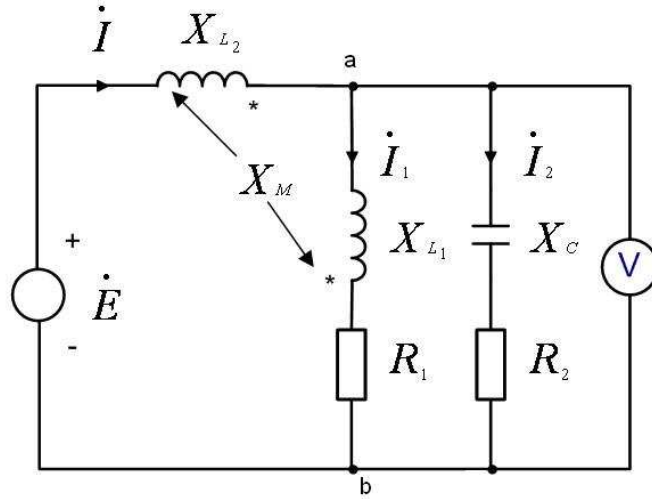
$$\begin{aligned} \dot{U}_{L2} &= \dot{I}_2 \cdot jX_{L2} + \dot{I}_1 \cdot jX_{12} - \dot{I}_3 \cdot jX_{23} = (-1 + j2) \cdot j4 + (1 - j7) \cdot j2 + (-2 + j9) \cdot j2 = \\ &= -j4 - 8 + j2 + 14 - j4 - 18 = -12 - j6 \text{ V} \end{aligned}$$

$$\dot{U}_{R2} = \dot{I}_2 \cdot R_2 = (-1 + j2) \cdot 2 = -2 + j4 \text{ V}$$

$$\dot{E} = \dot{U}_{L1} + \dot{U}_{L2} + \dot{U}_{R2} = 24 + j2 - 12 - j6 - 2 + j4 = 10 \text{ V}$$



11.3. Ako voltmetar u shemi na slici pokazuje $U_V = 100 \text{ V}$, a $X_{L1} = X_{L2} = X_C = X_M = R = 100 \Omega$, treba odrediti sve struje grana i nacrtati topografski dijagram.



$$\dot{U}_{ab} = 100 \angle 0^\circ \text{ V}$$

$$\dot{I}_2 = \frac{\dot{U}_{ab}}{R - jX_C} = \frac{100 \angle 0^\circ}{100 - j100} = \frac{100}{100 \cdot \sqrt{2} \angle -45^\circ} = \frac{\sqrt{2}}{2} \angle 45^\circ = 0,5 + j0,5 = 0,7071 \angle 45^\circ \text{ A}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = \dot{I}_1 + 0,5 + j0,5$$

$$\dot{U}_{ab} = \dot{I}_1 \cdot (R_1 + jX_{L1}) + \dot{I} \cdot jX_M = \dot{I}_1 \cdot (100 + j100) + \dot{I} \cdot j100$$

$$100 = \dot{I}_1 \cdot (100 + j100) + (\dot{I}_1 + 0,5 + j0,5) \cdot j100$$

$$100 = \dot{I}_1 \cdot (100 + j100) + \dot{I}_1 \cdot j100 + j50 - 50$$

$$150 - j50 = \dot{I}_1 \cdot (100 + j200)$$

$$\dot{I}_1 = \frac{150 - j50}{100 + j200} = 0,7071 \angle -81,87^\circ = 0,1 - j0,7 \text{ A}$$

$$\dot{I} = 0,1 - j0,7 + 0,5 + j0,5 = 0,6 - j0,2 = 0,63246 \angle -18,435^\circ \text{ A}$$

$$\dot{U}_{X_{L1}} = \dot{I}_1 \cdot jX_{L1} + \dot{I} \cdot jX_M = (0,1 - j0,7 + 0,6 - j0,2) \cdot j100 = (0,7 - j0,9) \cdot j100 = 90 + j70 \text{ V}$$

$$\dot{U}_{R1} = \dot{I}_1 \cdot R = (0,1 - j0,7) \cdot 100 = 10 - j70 \text{ V}$$

$$\dot{U}_{ab} = \dot{U}_{R1} + \dot{U}_{X_{L1}} = 10 - j70 + 90 + j70 = 100 \text{ V}$$

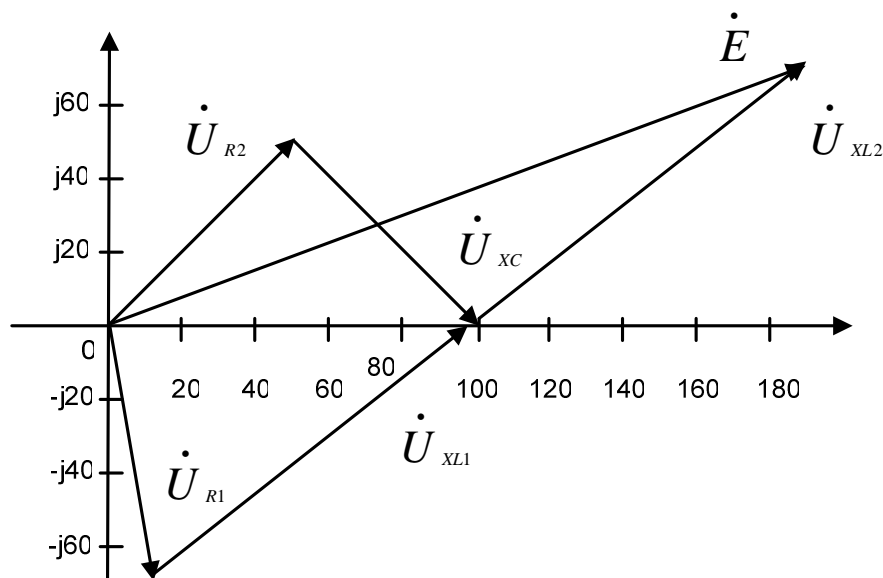
$$\dot{U}_{X_C} = \dot{I}_2 \cdot (-jX_C) = (0,5 + j0,5) \cdot (-j100) = 50 - j50 \text{ V}$$

$$\dot{U}_{R2} = \dot{I}_2 \cdot R_2 = (0,5 + j0,5) \cdot 100 = 50 + j50 \text{ V}$$

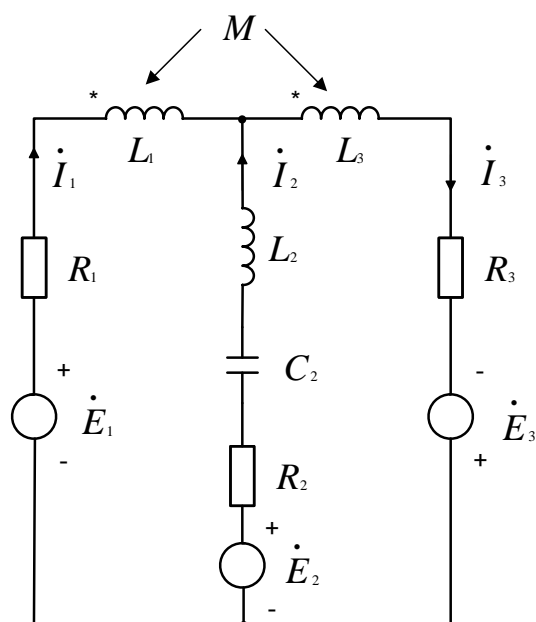
$$\dot{U}_{ab} = \dot{U}_{R2} + \dot{U}_{X_C} = 50 + j50 + 50 - j50 = 100 \text{ V}$$

$$\dot{U}_{X_{L2}} = \dot{I} \cdot jX_{L2} + \dot{I}_1 \cdot jX_M = (\dot{I} + \dot{I}_1) \cdot jX = (0,6 - j0,2 + 0,1 - j0,7) \cdot j100 = (0,7 - j0,9) \cdot j100 = 90 + j70 \text{ V}$$

$$\dot{E} = \dot{U}_{ab} + \dot{U}_{X_{L2}} = 100 + 90 + j70 = 190 + j70 \text{ V}$$



11.4. U mreži prema slici treba odrediti struje grana. Zadano: $R_1 = X_{L2} = X_{L3} = 80 \Omega$,
 $R_2 = R_3 = X_{C2} = X_M = 40 \Omega$, $X_{L1} = 60 \Omega$, $\dot{E}_1 = \dot{E}_2 = 100 \text{ V}$, $\dot{E}_3 = 200 \angle 120^\circ \text{ V}$.



$$\dot{E}_1 - \dot{E}_2 = 0 = \dot{I}_1 \cdot (R_1 + jX_{L1}) - \dot{I}_2 \cdot [R_2 + j \cdot (X_{L2} - X_{C2})] + \dot{I}_3 \cdot jX_M$$

$$\dot{E}_2 - \dot{E}_3 = \dot{I}_2 \cdot [R_2 + j \cdot (X_{L2} - X_{C2})] + \dot{I}_3 \cdot (R_3 + jX_{L3}) + \dot{I}_1 \cdot jX_M$$

$$\dot{I}_1 + \dot{I}_2 = \dot{I}_3$$

$$0 = \dot{I}_1 \cdot (80 + j60) - \dot{I}_2 \cdot (40 + j40) + \dot{I}_1 \cdot j40 + \dot{I}_2 \cdot j40$$

$$100 - 100 + j173,21 = \dot{I}_2 \cdot (40 + j40) + \dot{I}_1 \cdot (40 + j80) + \dot{I}_2 \cdot (40 + j80) + \dot{I}_1 \cdot j40$$

$$0 = \dot{I}_1 \cdot (80 + j100) - \dot{I}_2 \cdot 40 \Rightarrow \dot{I}_2 = \frac{80 + j100}{40} \cdot \dot{I}_1 = (2 + j2,5) \cdot \dot{I}_1$$

$$j173,21 = \dot{I}_1 \cdot (40 + j120) + \dot{I}_2 \cdot (80 + j120)$$

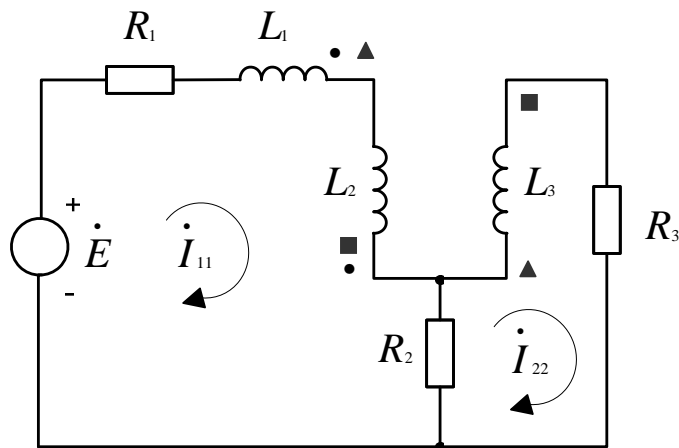
$$j173,21 = \dot{I}_1 \cdot (40 + j120) + (2 + j2,5) \cdot \dot{I}_1 \cdot (80 + j120) = \dot{I}_1 \cdot (40 + j120 - 140 + j440) = \\ = \dot{I}_1 \cdot (-100 + j560)$$

$$\dot{I}_1 = \frac{173,21 \angle 90^\circ}{568,86 \angle 100,12^\circ} = 0,3044 \angle -10,12^\circ = 0,3 - j0,053524 \text{ A}$$

$$\dot{I}_2 = 3,20156 \angle 51,34^\circ \cdot 0,3044 \angle -10,12^\circ = 0,9748 \angle 41,22^\circ = 0,73328 + j0,6423 \text{ A}$$

$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2 = 1,033 + j0,58877 = 1,189 \angle 29,68^\circ \text{ A}$$

11.5. Treba odrediti konturne struje na slici. Zadano: $\dot{E} = 100 \angle 0^\circ \text{ V}$, $R_1 = 10 \Omega$, $R_2 = 8 \Omega$, $R_3 = 4 \Omega$, $\omega L_1 = 6 \Omega$, $\omega L_2 = 12 \Omega$, $\omega L_3 = 10 \Omega$, $\omega M_{12} = 6 \Omega$, $\omega M_{31} = 5 \Omega$, $\omega M_{23} = 9 \Omega$, a oznake za istoimene priključnice su: • između L_1 i L_2 , ■ između L_2 i L_3 , ▲ između L_3 i L_1 .



$$\dot{E} = \dot{I}_{11} \cdot [R_1 + R_2 + j \cdot (X_{L1} + X_{L2} + 2 \cdot X_{12})] - \dot{I}_{22} \cdot [(R_2 + j \cdot (X_{13} - X_{23}))]$$

$$0 = -\dot{I}_{11} \cdot [R_2 + j \cdot (X_{13} - X_{23})] + \dot{I}_{22} \cdot (R_2 + R_3 + jX_{L3})$$

$$100 = \dot{I}_{11} \cdot [10 + 8 + j \cdot (6 + 12 + 2 + 6)] - \dot{I}_{22} \cdot [8 + j \cdot (5 - 9)]$$

$$0 = -\dot{I}_{11} \cdot [(8 + j \cdot (5 - 9))] - \dot{I}_{22} \cdot (8 + 4 + j10)$$

$$100 = \dot{I}_{11} \cdot (18 + j30) - \dot{I}_{22} \cdot (8 - j4)$$

$$0 = -\dot{I}_{11} \cdot (8 - j4) + \dot{I}_{22} \cdot (12 + j10)$$

$$\dot{I}_{11} = \frac{12 + j10}{8 - j4} \cdot \dot{I}_{22} = (0,7 + j1,6) \cdot \dot{I}_{22} = 1,7464 \angle 63,37^\circ \cdot \dot{I}_{22}$$

$$100 = (0,7 + j1,6) \cdot (18 + j30) \cdot \dot{I}_{22} - \dot{I}_{22} \cdot (8 - j4) = \dot{I}_{22} \cdot (-35,4 + j49,8 - 8 - j4)$$

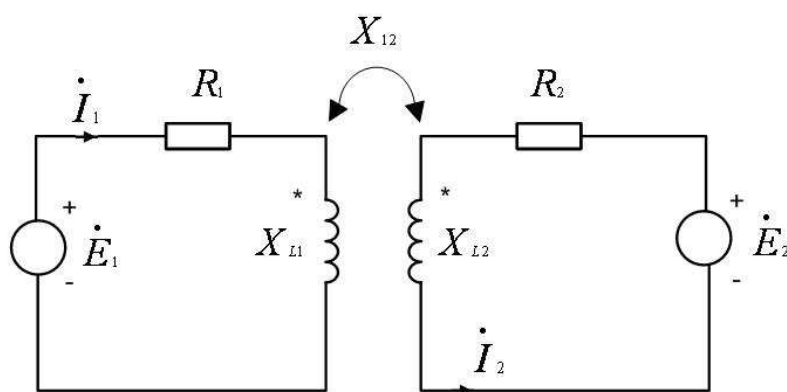
$$100 = \dot{I}_{22} \cdot (-43,4 + j53,8)$$

$$\dot{I}_{22} = \frac{100}{69,123 \angle 128,89^\circ} = 1,4467 \angle -128,89^\circ \text{ A}$$

$$\dot{I}_{11} = 1,7464 \angle 66,37^\circ \cdot 1,4467 \angle -128,89^\circ = 2,5265 \angle -62,52^\circ \text{ A}$$

11.6. Treba odrediti odnos napona \dot{E}_1 / \dot{E}_2 u shemi na slici za slučajeve kad je: a) $\dot{I}_1 = 0$;

b) $\dot{I}_2 = 0$. Zadano: $R_1 = 5 \Omega$, $R_2 = 2 \Omega$, $X_{L1} = 8 \Omega$, $X_{L2} = 2 \Omega$ i $X_{12} = 2 \Omega$.



$$\dot{E}_1 = \dot{I}_1 \cdot (5 + j8) + \dot{I}_2 \cdot j2$$

$$\dot{E}_2 = \dot{I}_1 \cdot j2 + \dot{I}_2 \cdot (2 + j2)$$

a) $\dot{I}_1 = 0$: $\dot{E}_1 = \dot{I}_2 \cdot j2$

$$\dot{E}_2 = \dot{I}_2 \cdot (2 + j2)$$

$$\frac{\dot{E}_1}{\dot{E}_2} = \frac{j2}{2 + j2} = 0,7071 \angle 45^\circ$$

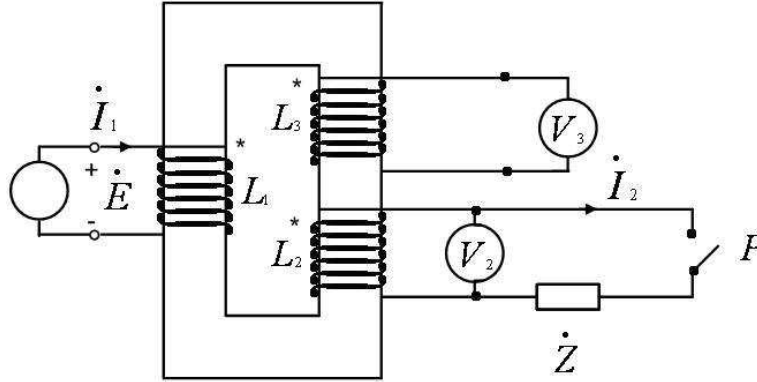
b) $\dot{I}_2 = 0$: $\dot{E}_2 = \dot{I}_1 \cdot j2$

$$\dot{E}_1 = \dot{I}_1 \cdot (5 + j8)$$

$$\frac{\dot{E}_1}{\dot{E}_2} = \frac{5 + j8}{j2} = 4,717 \angle -32^\circ$$

11.7. Na jezgri od izolatora namotana su tri svitka, prema slici. Odredite pokazivanja voltmetara V_2 i V_3 : a) pri otvorenom prekidaču P i b) pri zatvorenom prekidaču. Zadano:

$$\dot{E}_1 = 100 \angle 0^\circ \text{ V}, \quad X_{L1} = X_{L2} = X_{L3} = 100 \, \Omega, \quad k_{12} = k_{13} = 0,8, \quad k_{23} = 0,9 \quad \text{ i } \quad \dot{Z} = 10 \, \Omega.$$



$$\text{a) } \dot{I}_1 = \frac{\dot{E}}{jX_{L1}} = \frac{100}{100 \angle 90^\circ} = 1 \angle -90^\circ \text{ A}$$

$$X_{12} = X_{13} = 0,8 \cdot \sqrt{100 \cdot 100} = 80 \, \Omega$$

$$\dot{U}_{V2} = \dot{I}_1 \cdot jX_{M12} = \dot{I}_1 \cdot jX_{M13} = 1 \angle -90^\circ \cdot 80 \angle 90^\circ = 80 \text{ V} = \dot{U}_{V3}$$

$$\text{b) } \dot{E} = \dot{I}_1 \cdot jX_{L1} - \dot{I}_2 \cdot jX_{M12} \quad (1)$$

$$\dot{I}_1 \cdot jX_{M12} = \dot{I}_2 \cdot (\dot{Z} + jX_{L2}) \quad (2)$$

$$(2) \rightarrow \dot{I}_2 = \frac{jX_{M12}}{\dot{Z} + jX_{L2}} \cdot \dot{I}_1 = \frac{80 \angle 90^\circ \cdot \dot{I}_1}{10 + j100} = \frac{80 \angle 90^\circ \cdot \dot{I}_1}{104,5 \angle 84,29^\circ} = 0,796 \angle 5,71^\circ \cdot \dot{I}_1$$

$$(1) \rightarrow 100 = \dot{I}_1 \cdot j100 + 0,796 \angle 5,71^\circ \cdot \dot{I}_1 \cdot 80 \angle -90^\circ = \dot{I}_1 \cdot (j100 + 63,682 \angle -84,29^\circ)$$

$$100 = \dot{I}_1 \cdot (j100 + 6,337 - j63,366) = \dot{I}_1 \cdot 37,1776 \angle 80,186^\circ$$

$$\dot{I}_1 = \frac{100}{37,1776 \angle 80,186^\circ} = 2,69 \angle -80,186^\circ \text{ A}$$

$$\dot{I}_2 = 0,796 \angle 5,71^\circ \cdot 2,69 \angle -80,186^\circ = 2,141 \angle -74,475^\circ \text{ A}$$

$$\dot{U}_{V2} = \dot{I}_2 \cdot \dot{Z} = 2,141 \angle -74,475^\circ \cdot 10 = 21,41 \angle -74,475^\circ \text{ V} \Rightarrow U_{V2} = 21,41 \text{ V}$$

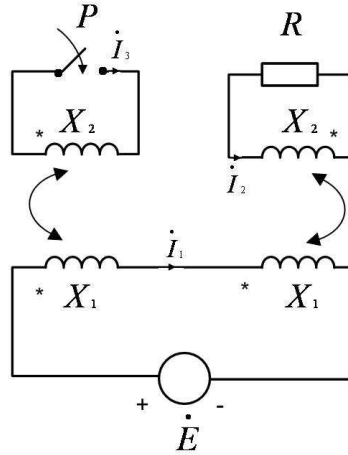
$$X_{M23} = k_{23} \cdot \sqrt{X_{L2} \cdot X_{L3}} = 0,9 \cdot \sqrt{100^2} = 90 \, \Omega$$

$$\dot{U}_{V3} = \dot{I}_1 \cdot jX_{M13} - \dot{I}_2 \cdot jX_{M23} = 2,69 \angle -80,186^\circ \cdot 80 \angle 90^\circ + 2,141 \angle -74,475^\circ \cdot 90 \angle -90^\circ =$$

$$= 215,183 \angle 9,8135^\circ + 192,7 \angle -164,476^\circ = 212,03 + j36,676 - 185,673 - j51,576 = 26,361 - j14,9$$

$$\dot{U}_{V3} = 30,28 \angle -29,5^\circ \text{ V} \Rightarrow U_{V3} = 30,28 \text{ V}$$

11.8. Treba odrediti snagu na otporu R u shemi na slici , a) pri otvorenom prekidaču, b) pri zatvorenom prekidaču. Zadano: $\dot{E} = 200 \angle 0^\circ \text{ V}$, $R = 100 \, \Omega$, $X_1 = 24 \, \Omega$, $X_2 = 150 \, \Omega$, faktor induktivne veze između svitaka je $k = 1$.



a) $X_M = k \cdot \sqrt{X_1 \cdot X_2} = 1 \cdot \sqrt{24 \cdot 150} = 60 \, \Omega$

$$\dot{E} = \dot{I}_1 \cdot j2X_1 - \dot{I}_2 \cdot jX_M \quad (1)$$

$$0 = -\dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot (R + jX_2) \quad (2)$$

$$(2) \rightarrow \dot{I}_1 = \frac{R + jX_2}{jX_M} \cdot \dot{I}_2 = \frac{100 + j150}{j60} \cdot \dot{I}_2 = \frac{180,28 \angle 56,31^\circ}{60 \angle 90^\circ} \cdot \dot{I}_2$$

$$\dot{I}_1 = 3,005 \angle -33,69^\circ \cdot \dot{I}_2 = (2,5 - j1,6) \cdot \dot{I}_2$$

$$(1) \rightarrow 200 = (2,5 - j1,6) \cdot \dot{I}_2 \cdot j48 - \dot{I}_2 \cdot j60$$

$$200 = \dot{I}_2 \cdot (80 + j120 - j60) = \dot{I}_2 \cdot (80 + j60)$$

$$\dot{I}_2 = \frac{200}{80 + j60} = \frac{200}{100 \angle 36,87^\circ} = 2 \angle -36,87^\circ \Rightarrow P = I_2^2 R = 2^2 \cdot 100 = 400 \text{ W}$$

b) (1) $\dot{E} = \dot{I}_1 \cdot j2X_1 - \dot{I}_2 \cdot jX_M - \dot{I}_3 \cdot jX_M$

$$(2) 0 = -\dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot (R + jX_2) \Rightarrow \dot{I}_1 = \frac{R + jX_2}{jX_M} \cdot \dot{I}_2 = (2,5 - j1,6) \cdot \dot{I}_2$$

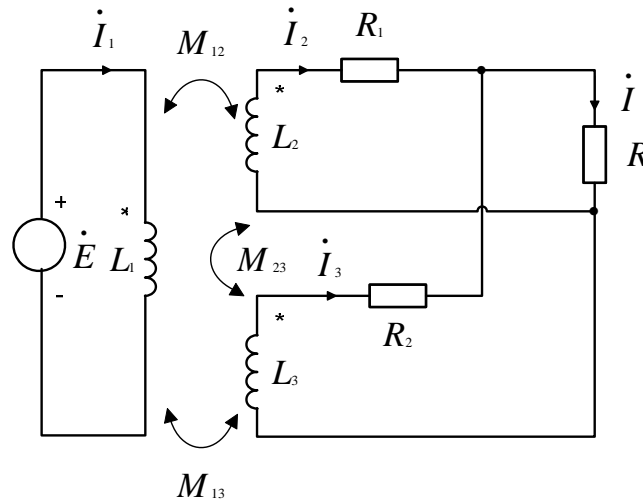
$$(3) 0 = -\dot{I}_1 \cdot jX_M + \dot{I}_3 \cdot jX_2 \Rightarrow \dot{I}_3 = \frac{X_M}{X_2} \cdot \dot{I}_1 = \frac{60}{150} \cdot (2,5 - j1,6) \cdot \dot{I}_2 = (1 - j0,6) \cdot \dot{I}_2$$

$$(1) \rightarrow 200 = (2,5 - j1,6) \cdot \dot{I}_2 \cdot j48 - \dot{I}_2 \cdot j60 - (1 - j0,6) \cdot \dot{I}_2 \cdot j60$$

$$200 = \dot{I}_2 \cdot (80 + j120 - j60 - j60 - 40) = 40 \cdot \dot{I}_2$$

$$\dot{I}_2 = \frac{200}{40} = 5 \text{ A} \Rightarrow P = I_2^2 \cdot R = 5^2 \cdot 100 = 2500 \text{ W}$$

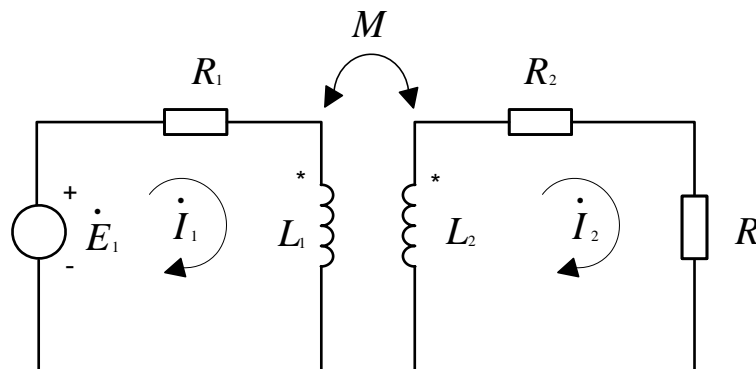
11.9. U shemi na slici dva namota transformatora rade paralelno, treba odrediti izraze za struje \dot{I}_2 , \dot{I}_3 i \dot{I} .



$$\begin{aligned}\dot{I} &= \dot{I}_2 + \dot{I}_3 \\ jX_{L1} \cdot \dot{I}_1 - jX_{M12} \cdot \dot{I}_2 - jX_{M13} \cdot \dot{I}_3 &= \dot{E} \\ -jX_{M12} \cdot \dot{I}_1 + \dot{I}_2 \cdot (jX_{L2} + R_1) + jX_{M23} \cdot \dot{I}_3 + \dot{I} \cdot R &= 0 \\ -jX_{M13} \cdot \dot{I}_1 + jX_{M23} \cdot \dot{I}_2 + \dot{I}_3 \cdot (R_2 + jX_{L3}) + \dot{I} \cdot R &= 0\end{aligned}$$

11.10. Transformator s jezgrom od izolatora prema slici, ima sljedeće karakteristike: $E = 20 \text{ V}$, $N_1 = 50$ zavoja, $N_2 = 500$ zavoja, $L_1 = 0,003 \text{ H}$, $L_2 = 0,3 \text{ H}$, $R_1 = 2 \Omega$, $R_2 = 100 \Omega$, $R = 200 \Omega$, $K_1 = K_2 = K_3 = 0,5$, $\omega = 1000 \text{ rad/s}$. Treba:

- odrediti struje \dot{I}_1 i \dot{I}_2 ;
- nacrtati fazorski dijagram napona i struja transformatora sa svim tokovima;
- izračunati ulaznu impedanciju;
- nacrtati nadomjesnu shemu transformatora.



$$k = \sqrt{K_1 \cdot K_2} = \sqrt{0,5^2} = 0,5 \Omega$$

$$X_1 = 3 \Omega$$

$$X_2 = 300 \Omega$$

$$X_M = \sqrt{3 \cdot 300} = 15 \Omega$$

$$a) \quad \dot{E}_1 = \dot{I}_1 \cdot (R_1 + jX_1) - \dot{I}_2 \cdot jX_M = 20 = \dot{I}_1 \cdot (2 + j3) - \dot{I}_2 \cdot j15$$

$$0 = -\dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot (R + R_2 + jX_2) = -\dot{I}_1 \cdot j15 + \dot{I}_2 \cdot (300 + j300)$$

$$\dot{I}_1 = \frac{300 + j300}{j15} \cdot \dot{I}_2 = \frac{300 \cdot \sqrt{2} \angle 45^\circ}{15 \angle 90^\circ} \cdot \dot{I}_2 = (20 + j20) \cdot \dot{I}_2$$

$$20 = (20 - j20) \cdot \dot{I}_2 \cdot (2 + j3) - \dot{I}_2 \cdot j15$$

$$20 = (40 + 60 + j60 - j40 - j15) \cdot \dot{I}_2 = (100 - j5) \cdot \dot{I}_2$$

$$\dot{I}_2 = \frac{20}{100,125 \angle 2,862^\circ} = 0,19975 \angle -2,862^\circ A$$

$$\dot{I}_1 = 20 \cdot \sqrt{2} \angle -45^\circ \cdot 0,19975 \angle -2,862^\circ = 5,6498 \angle -47,862^\circ A$$

$$b) \quad \dot{U}_{R1} = \dot{I}_1 \cdot R_1 = 11,3 \angle -47,862^\circ = 7,58 - j8,38$$

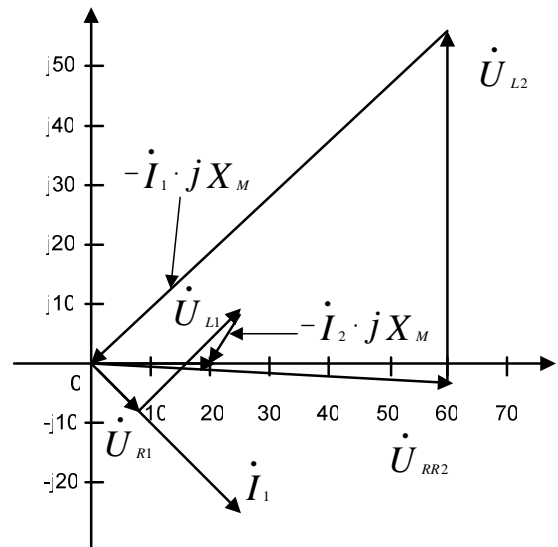
$$\dot{U}_{L1} = \dot{I}_1 \cdot jX_1 = 16,95 \angle 42,138^\circ = 12,57 + j11,37$$

$$\frac{-\dot{I}_2 \cdot jX_M = 3 \angle -92,862^\circ = -0,15 - j3}{\Sigma \quad 20 + j20}$$

$$\dot{U}_{RR2} = \dot{I}_2 \cdot (R_2 + R) = 60 \angle -2,862^\circ = 59,93 - j3$$

$$\dot{U}_{L2} = \dot{I}_2 \cdot jX_2 = 60 \angle 87,138^\circ = 3 + j59,93$$

$$\frac{-\dot{I}_1 \cdot jX_M = 84,75 \angle -137,862^\circ = -62,85 - j56,86}{\Sigma \quad 0}$$



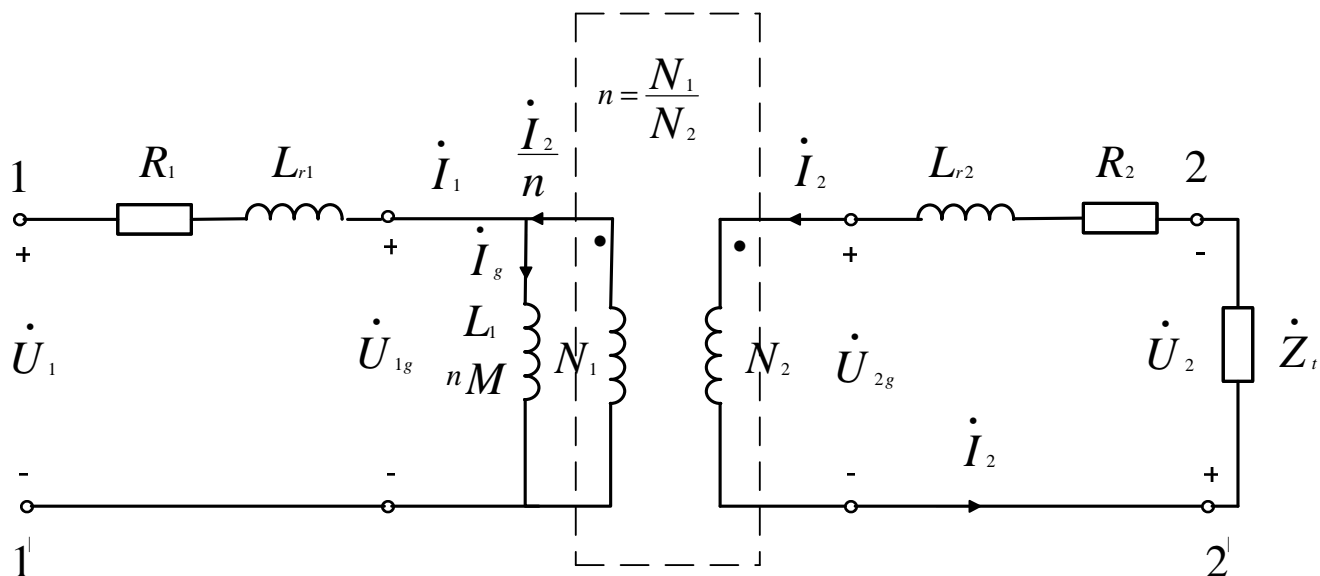
$$c) \quad \dot{Z}_{ul} = \frac{\dot{E}}{\dot{I}_1} = \frac{20}{5,6498 \angle -47,862^\circ} = 3,54 \angle 47,862^\circ \Omega$$

$$d) \quad L_{r1} = L_1 \cdot (1 - K_1) = 0,003 \cdot (1 - 0,5) = 1,5 mH$$

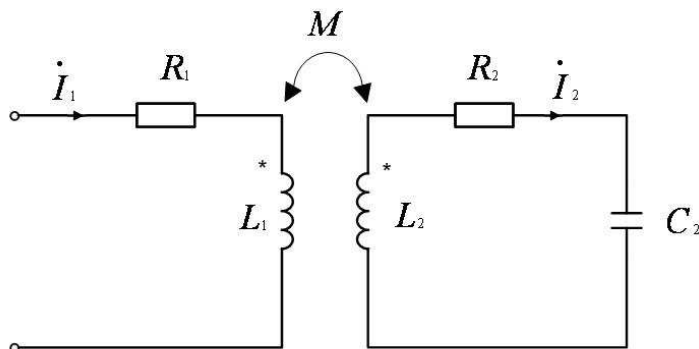
$$M' = nM = K_1 \cdot L_1 = 0,5 \cdot 0,003 = 1,5 mH$$

$$L_{r2} = L_2 \cdot (1 - K_2) = 0,3 \cdot 0,5 = 0,15 H$$

$$n = \frac{N_1}{N_2} = \frac{50}{500} = 0,1$$



- 11.11. Treba izračunati ulaznu impedanciju u shemi na slici za frekvencije: $f_1 = 997 \text{ kHz}$; $f_2 = 1000 \text{ kHz}$, $f_3 = 1003 \text{ kHz}$; $f_4 = 1006 \text{ kHz}$. Zadano je:
 $R_1 = 10 \Omega$, $L_1 = 65 \mu\text{H}$, $M = 49 \mu\text{H}$, $R_2 = 25 \Omega$, $L_2 = 150 \mu\text{H}$, $C_2 = 169 \text{ pF}$.



$$\dot{E} = \dot{I}_1 \cdot (R_1 + jX_1) - j\dot{I}_2 \cdot X_M$$

$$0 = -\dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot [R_2 + j \cdot (X_2 - X_{C2})] \Rightarrow \dot{I}_2 = \frac{jX_M \cdot \dot{I}_1}{R_2 + j \cdot (X_2 - X_{C2})}$$

$$\dot{E} = \dot{I}_1 \cdot (R_1 + jX_1) - X_M \cdot \frac{jX_M}{R_2 + j \cdot (X_2 - X_{C2})} \cdot \dot{I}_1$$

$$\dot{E} = \dot{I}_1 \cdot \left[R_1 + jX_1 + \frac{X_M^2}{R_2 + j \cdot (X_2 - X_{C2})} \cdot \frac{R_2 - j \cdot (X_2 - X_{C2})}{R_2 - j \cdot (X_2 - X_{C2})} \right]$$

$$\frac{\dot{E}}{\dot{I}} = R_1 + jX_1 + \frac{R_2 \cdot X_M^2 - j \cdot (X_2 - X_{C2}) \cdot X_M^2}{R_2^2 + (X_2 - X_{C2})^2}$$

$$\dot{Z}_{ul} = R_1 + \frac{R_2 \cdot X_M^2}{R_2^2 + (X_2 - X_{C2})^2} + j \cdot \left[X_1 - \frac{(X_2 - X_{C2}) \cdot X_M^2}{R_2^2 + (X_2 - X_{C2})^2} \right] = R + jX$$

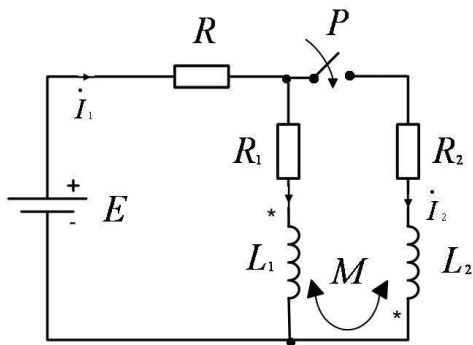
$$f_0 = \frac{1}{\sqrt{L_2 \cdot C_2}} = 999,61 \text{ kHz}$$

$f \text{ [Hz]}$	$997 \cdot 10^3$	$1000 \cdot 10^3$	$1003 \cdot 10^3$	$1006 \cdot 10^3$
$\omega \text{ [rad/s]}$	$6,264336 \cdot 10^6$	$6,283185 \cdot 10^6$	$6,302 \cdot 10^6$	$6,32088 \cdot 10^6$
$X_1 = \omega L_1 \text{ [\Omega]}$	407,18	408,407	409,63	410,857
$X_M = \omega M \text{ [\Omega]}$	306,95	307,876	308,8	309,723
$X_2 = \omega L_2 \text{ [\Omega]}$	939,65	942,478	945,3	948,133
$X_{C2} = \frac{1}{\omega C_2} \text{ [\Omega]}$	944,58	941,745	938,93	936,128
$R \text{ [\Omega]}$	3637,8	3798,25	3591,29	3128,2
$X \text{ [\Omega]}$	1122,38	297,4	-503,85	-1086,405
$\dot{Z} \text{ [\Omega]}$	$3807 \mid \underline{17,15^\circ}$	$3809,9 \mid \underline{4,477^\circ}$	$3626,5 \mid \underline{-8^\circ}$	$3311,5 \mid \underline{-19,15^\circ}$

11.12. Treba odrediti ukupnu magnetsku energiju u stacionarnom stanju u shemi na slici:

a) prije zatvaranja prekidača P ; b) poslije zatvaranja prekidača P .

Zadano: $E = 30 \text{ V}$, $R = R_2 = 2 \text{ }\Omega$, $R_1 = 3 \text{ }\Omega$, $L_1 = L_2 = 8 \text{ mH}$, $k = 0,8$.



$$M = k \cdot \sqrt{L_1 \cdot L_2} = 0,8 \cdot \sqrt{(8 \cdot 10^{-3})^2} = 6,4 \text{ mH}$$

$$a) \quad I_1 = \frac{E}{R + R_1} = \frac{30}{2 + 3} = \frac{30}{5} = 6 \text{ A}$$

$$W = \frac{L_1 \cdot I_1^2}{2} = \frac{8 \cdot 10^{-3} \cdot 6^2}{2} = 0,144 \text{ J}$$

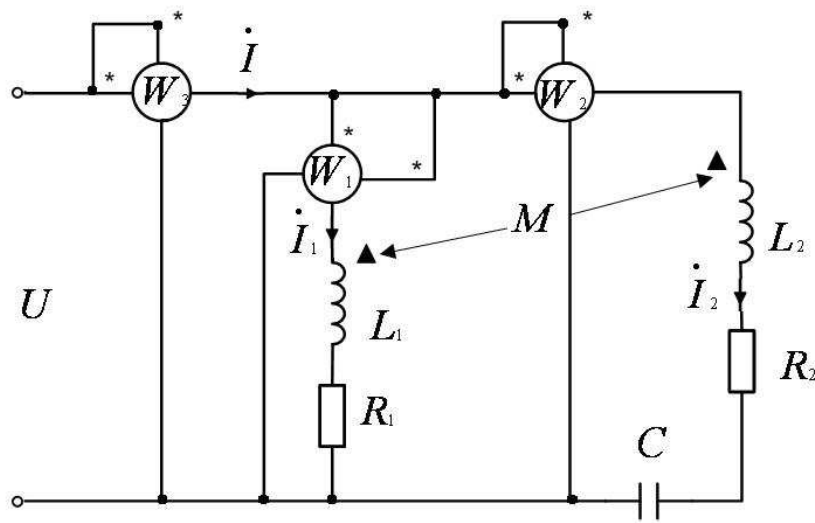
$$b) \quad R_u = R + \frac{R_1 \cdot R_2}{R_1 + R_2} = 2 + \frac{3 \cdot 2}{3 + 2} = 3,2 \text{ }\Omega$$

$$I_1 = \frac{E}{R_u} = \frac{30}{3,2} = 9,375 \text{ A}$$

$$U_p = I_1 \cdot 1,2 = 11,25 \text{ V} : \quad I_1 - I_2 = \frac{U_p}{R_1} = \frac{11,25}{3} = 3,75 \text{ A} \quad I_2 = \frac{U_p}{R_2} = 5,625 \text{ A}$$

$$\begin{aligned}
 W &= \frac{L_1 \cdot (I_1 - I_2)^2}{2} + \frac{L_2 \cdot I_2^2}{2} - M \cdot I_2 \cdot (I_1 - I_2) = \\
 &= \frac{8 \cdot 10^{-3} \cdot 3,75^2}{2} + \frac{8 \cdot 10^{-3} \cdot 5,625^2}{2} - 6,4 \cdot 10^{-3} \cdot 5,625 \cdot 3,75 = \\
 &= 0,05625 + 0,1265625 - 0,135 = 0,0478125 \text{ J}
 \end{aligned}$$

11.13. Treba odrediti pokazivanje vatmetra na slici. Zadano: $\dot{U} = 150 + j150 \text{ V}$, $R_1 = 5 \Omega$, $R_2 = 10 \Omega$, $\omega L_1 = 15 \Omega$, $\omega M = 10 \Omega$, $1/\omega C = 10 \Omega$, $\omega L_2 = 15 \Omega$.



$$\begin{aligned}
 \dot{U} &= \dot{I}_1 \cdot (R_1 + jX_{L1}) + \dot{I}_2 \cdot jX_M \\
 \dot{U} &= \dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot [R_2 + j \cdot (X_{L2} - X_C)] \\
 150 \cdot \sqrt{2} \angle 45^\circ &= \dot{I}_1 \cdot (5 + j15) + \dot{I}_2 \cdot j10 \\
 150 \cdot \sqrt{2} \angle 45^\circ &= \dot{I}_1 \cdot j10 + \dot{I}_2 \cdot [10 + j \cdot (15 - 10)] = \dot{I}_1 \cdot j10 + \dot{I}_2 \cdot (10 + j5) \\
 \dot{I}_2 &= \frac{150 \cdot \sqrt{2} \angle 45^\circ}{10 \angle 90^\circ} - \frac{5 + j15}{10 \angle 90^\circ} \cdot \dot{I}_1 = 15 \cdot \sqrt{2} \angle -45^\circ - (1,5 - j0,5) \cdot \dot{I}_1 \\
 150 \cdot \sqrt{2} \angle 45^\circ &= \dot{I}_1 \cdot j10 + (10 + j5) \cdot \left[15 \cdot \sqrt{2} \angle -45^\circ - (1,5 - j0,5) \cdot \dot{I}_1 \right] = \\
 &= \dot{I}_1 \cdot j10 + 11,1803 \angle 26,565^\circ \cdot \left[15 \cdot \sqrt{2} \angle -45^\circ + 1,5811 \angle 161,565^\circ \cdot \dot{I}_1 \right] = \\
 &= \dot{I}_1 \cdot j10 + 237,17 \angle -18,435^\circ + 17,678 \angle 188,13^\circ \cdot \dot{I}_1 \\
 150 + j150 &= \dot{I}_1 \cdot j10 + 225 - j75 + (-17,5 - j2,5) \cdot \dot{I}_1 \\
 \dot{I}_1 \cdot (-17,5 + j7,5) &= 150 - 225 + j150 + j75 = -75 + j225
 \end{aligned}$$

$$\dot{I}_1 = \frac{-75 + j225}{-17,5 + j7,5} = \frac{237,17 \angle 108,435^\circ}{19,0394 \angle 156,8^\circ} = 12,4568 \angle -48,366^\circ = 8,27586 - j9,31034 \text{ A}$$

$$\dot{I}_2 = 15 + j15 + 1,5811 \angle 161,565^\circ \cdot 12,4568 \angle -48,366^\circ = 7,24138 + j3,103448 = 7,8784 \angle 23,2^\circ \text{ A}$$

$$P_{W1} = \operatorname{Re}\{\dot{U} \cdot \dot{I}_1^*\} = \operatorname{Re}\{150 \cdot \sqrt{2} \angle 45^\circ \cdot 12,4568 \angle 48,366^\circ\} = \operatorname{Re}\{2642,49 \angle 93,366^\circ\} = \\ = \operatorname{Re}\{-155,17 + j2637,9\} = -155,17 \text{ W}$$

$$P_{W2} = \operatorname{Re}\{\dot{U} \cdot \dot{I}_2^*\} = \operatorname{Re}\{150 \cdot \sqrt{2} \angle 45^\circ \cdot 7,8784 \angle -23,2^\circ\} = \operatorname{Re}\{1671,258 \angle 21,8^\circ\} = \\ = \operatorname{Re}\{1551,72 + j620,69\} = 1551,72 \text{ W}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2 = 8,27586 - j9,31034 + 7,24138 + j3,103448 = 15,5172 - j6,2096 = 16,7126 \angle -21,8^\circ \text{ A}$$

$$P_{W3} = \operatorname{Re}\{\dot{U} \cdot \dot{I}^*\} = \operatorname{Re}\{150 \cdot \sqrt{2} \angle 45^\circ \cdot 16,7126 \angle 21,8^\circ\} = \operatorname{Re}\{3545,273 \angle 66,8^\circ\} = \\ = \operatorname{Re}\{1396,55 + j3258,62\} = 1396,55 \text{ W}$$

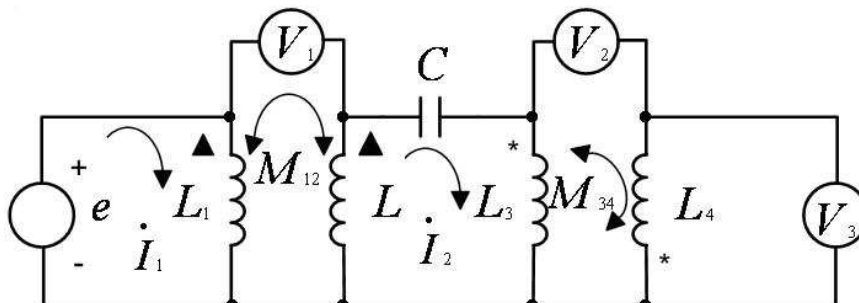
Ili: $\dot{Z}_1 = \frac{\dot{U}}{\dot{I}_1} = \frac{150 \cdot \sqrt{2} \angle 45^\circ}{12,4568 \angle -48,366^\circ} = 17,029 \angle 93,366^\circ = -1 + j17$

$$P_{W1} = I_1^2 R_1' = 12,4568^2 \cdot (-1) = -155,17 \text{ W}$$

$$\dot{Z}_2 = \frac{\dot{U}}{\dot{I}_2} = \frac{150 \cdot \sqrt{2} \angle 45^\circ}{7,8784 \angle 23,2^\circ} = 26,926 \angle 21,8^\circ = 25 + j10$$

$$P_{W2} = I_2^2 + R_2' = 7,8784^2 \cdot 25 = 1551,72 \text{ W}$$

11.14. U shemi na slici treba odrediti što pokazuju voltmetri. Zadano: $e = 120\sqrt{2} \cdot \sin 4000t \text{ V}$,
 $L_1 = L_3 = 20 \text{ mH}$, $L_2 = L_4 = 25 \text{ mH}$, $M_{12} = M_{34} = 10 \text{ mH}$, $C = 2,5 \text{ }\mu\text{F}$.



$$\omega = 4 \cdot 10^3 \text{ rad/s}$$

$$X_1 = X_3 = 4 \cdot 10^3 \cdot 20 \cdot 10^{-3} = 80 \, \Omega$$

$$X_2 = X_4 = 4 \cdot 10^3 \cdot 25 \cdot 10^{-3} = 100 \, \Omega$$

$$X_{12} = X_{34} = 4 \cdot 10^3 \cdot 10 \cdot 10^{-3} = 40 \, \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{4 \cdot 10^3 \cdot 2,5 \cdot 10^{-6}} = 100 \, \Omega$$

$$\dot{E} = \dot{I}_1 \cdot jX_1 - \dot{I}_2 \cdot jX_{12}$$

$$0 = -\dot{I}_1 \cdot jX_{12} + \dot{I}_2 \cdot j \cdot (X_2 + X_3 - X_C)$$

$$120 = \dot{I}_1 \cdot j80 - \dot{I}_2 \cdot j40$$

$$0 = -\dot{I}_1 \cdot j40 + \dot{I}_2 \cdot j(100 + 80 - 100) = \dot{I}_2 \cdot j80 - \dot{I}_1 \cdot j40$$

$$\dot{I}_1 = \frac{j80}{j40} \cdot \dot{I}_2 = 2 \cdot \dot{I}_2$$

$$120 = 2 \cdot \dot{I}_2 \cdot j80 - \dot{I}_2 \cdot j40 = \dot{I}_2 \cdot j(160 - 40) = j120 \cdot \dot{I}_2$$

$$\dot{I}_2 = \frac{120}{j120} = 1 \angle -90^\circ = -j \, A \quad \Rightarrow \quad \dot{I}_1 = -2j \, A$$

$$\begin{aligned} \dot{U}_{V1} &= \dot{I}_1 \cdot jX_1 - \dot{I}_2 \cdot jX_{12} - \dot{I}_1 \cdot jX_{12} + \dot{I}_2 \cdot jX_2 = \dot{I}_1 \cdot j \cdot (X_1 - X_{12}) + \dot{I}_2 \cdot j \cdot (X_2 - X_{12}) = \\ &= -2j \cdot j \cdot (80 - 40) - j \cdot j \cdot (100 - 40) = 2 \cdot 40 + 60 = 140 \, V \end{aligned}$$

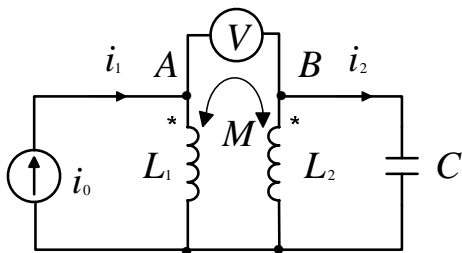
$$\dot{U}_{V2} = \dot{I}_2 \cdot jX_3 + \dot{I}_2 \cdot jX_{34} = -j \cdot j80 - j \cdot j \cdot 40 = 80 + 40 = 120 \, V$$

↓

$$\dot{U}_{V3}$$

11.15. U shemi na slici treba odrediti napon u_{AB} i pokazivanje voltmetra. Zadano je:

$$i_0 = 2 + 4 \sin \omega t + 2 \sin(2\omega t + 30^\circ) \, A, \quad \omega L_1 = 5 \, \Omega, \quad \omega L_2 = 10 \, \Omega, \quad \omega M = 2,5 \, \Omega, \quad 1/\omega C = 60 \, \Omega.$$



$$a) n = 1: \quad X_1 = 5 \, \Omega$$

$$X_2 = 10 \, \Omega$$

$$X_M = 2,5 \, \Omega$$

$$X_C = 60 \, \Omega$$

$$\dot{E} = \dot{I}_1 \cdot jX_1 - \dot{I}_2 \cdot jX_M$$

$$0 = -\dot{I}_1 \cdot jX_M + \dot{I}_2 \cdot j \cdot (X_2 - X_C)$$

$$\dot{E}_1 = 2 \cdot \sqrt{2} \cdot j5 - \dot{I}_2 \cdot j2,5$$

$$0 = -2 \cdot \sqrt{2} \cdot j2,5 + \dot{I}_2 \cdot j \cdot (10 - 60) / : j \quad \Rightarrow \quad \dot{I}_2 = \frac{5 \cdot \sqrt{2}}{-50} = -0,1 \cdot \sqrt{2} \text{ A}$$

$$\dot{E}_1 = 2\sqrt{2} \cdot j5 + 0,1 \cdot \sqrt{2} \cdot j2,5 = j \cdot \sqrt{2} \cdot (10 + 0,25) = j10,25 \cdot \sqrt{2} \text{ V}$$

$$\begin{aligned} \dot{U}_{AB1} &= \dot{E}_1 + \dot{I}_2 \cdot jX_2 - \dot{I}_1 \cdot jX_M = j10,25 \cdot \sqrt{2} - 0,1 \cdot \sqrt{2} \cdot j10 - 2 \cdot \sqrt{2} \cdot j2,5 = \\ &= j\sqrt{2} \cdot (10,25 - 1 - 5) = j4,25 \cdot \sqrt{2} \text{ V} \end{aligned}$$

b) $n = 2$:

$$\begin{aligned} X_1 &= 10 \, \Omega \\ X_2 &= 20 \, \Omega \\ X_M &= 5 \, \Omega \\ X_C &= 30 \, \Omega \end{aligned}$$

$$\dot{E}_2 = \sqrt{2} \mid \underline{30^\circ} \cdot j10 - \dot{I}_2 \cdot j5$$

$$0 = -\sqrt{2} \mid \underline{30^\circ} \cdot j5 + \dot{I}_2 \cdot j \cdot (20 - 30) / : j \quad \Rightarrow \quad \dot{I}_2 = \frac{5 \cdot \sqrt{2} \mid \underline{30^\circ}}{-10} = -0,5 \cdot \sqrt{2} \mid \underline{30^\circ} \text{ A}$$

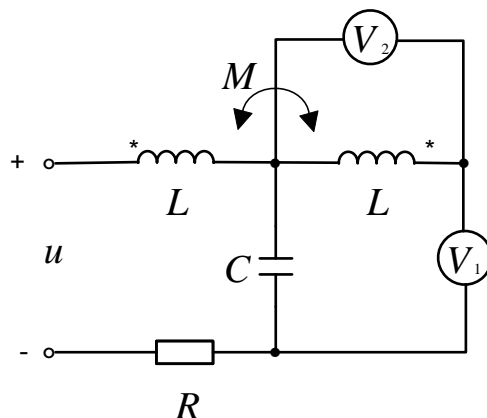
$$\dot{E}_2 = j \cdot \sqrt{2} \mid \underline{30^\circ} \cdot 10 + 0,5 \cdot \sqrt{2} \mid \underline{30^\circ} \cdot j5 = j \cdot \sqrt{2} \mid \underline{30^\circ} \cdot (10 + 2,5) = 12,5 \cdot \sqrt{2} \mid \underline{120^\circ} \text{ V}$$

$$\begin{aligned} \dot{U}_{AB2} &= \dot{E}_2 + \dot{I}_2 \cdot jX_2 - \dot{I}_1 \cdot jX_M = 12,5 \cdot \sqrt{2} \mid \underline{120^\circ} - 0,5 \cdot \sqrt{2} \mid \underline{30^\circ} \cdot j20 - \sqrt{2} \mid \underline{30^\circ} \cdot j5 = \\ &= \sqrt{2} \cdot (12,5 - 10 - 5) \mid \underline{120^\circ} = 2,5 \cdot \sqrt{2} \mid \underline{-60^\circ} = 3,53 \mid \underline{-60^\circ} \text{ V} \end{aligned}$$

$$u_{AB} = 8,5 \cdot \sin(\omega t + 90^\circ) + 5 \cdot \sin(2\omega t - 60^\circ) \text{ V}$$

$$U_V = \sqrt{(4,25 \cdot \sqrt{2})^2 + (2,5 \cdot \sqrt{2})^2} = 6,973 \text{ V}$$

11.16. Treba odrediti što pokazuju voltmetri na slici. Zadano je: $R = \omega L = 1/\omega C = 4 \, \Omega$, $\omega M = 3 \, \Omega$, $u = 100 \cdot \sin \omega t + 50 \cdot \sin 2\omega t \text{ V}$.



a) $n = 1$:

$$\begin{aligned} R &= 4 \, \Omega & X_C &= 4 \, \Omega \\ X_L &= 4 \, \Omega & X_M &= 3 \, \Omega \end{aligned}$$

$$\dot{U}_1 = \frac{100}{\sqrt{2}} V$$

$$\dot{I}_1 = \frac{\dot{U}}{R + j \cdot (X_L - X_C)}$$

$$\dot{I}_1 = \frac{100}{\sqrt{2} \cdot [4 + j \cdot (4 - 4)]} = \frac{25}{\sqrt{2}} = 12,5 \cdot \sqrt{2} \text{ A}$$

$$\dot{U}_{V21} = \dot{I}_1 \cdot jX_M = 12,5 \cdot \sqrt{2} \cdot j3 = 37,5 \cdot \sqrt{2} \cdot \underline{90^\circ} V$$

$$\dot{U}_{V11} = \dot{I}_1 \cdot jX_M + \dot{I}_1 \cdot (-jX_C) = 37,5 \cdot \sqrt{2} \cdot \underline{90^\circ} + 12,5 \cdot \sqrt{2} \cdot 4 \cdot \underline{-90^\circ} = 12,5 \cdot \sqrt{2} \cdot \underline{-90^\circ} V$$

b) $n = 2$: $R = 4 \Omega$ $X_C = 2 \Omega$
 $X_L = 8 \Omega$ $X_M = 6 \Omega$

$$\dot{U}_2 = \frac{50}{\sqrt{2}} V$$

$$\dot{I}_2 = \frac{50}{\sqrt{2} \cdot [4 + j \cdot (8 - 2)]} = \frac{50}{\sqrt{2} \cdot (4 + j6)} = 4,903 \cdot \underline{-56,31^\circ} A$$

$$\dot{U}_{V22} = \dot{I}_2 \cdot jX_M = 4,903 \cdot \underline{-56,31^\circ} \cdot 6 \cdot \underline{90^\circ} = 29,417 \cdot \underline{33,69^\circ} = 24,477 + j16,318 V$$

$$\begin{aligned} \dot{U}_{V12} &= \dot{I}_2 \cdot jX_M + \dot{I}_2 \cdot (-jX_C) = 24,477 + j16,318 + 4,903 \cdot \underline{-56,31^\circ} \cdot 2 \cdot \underline{-90^\circ} = \\ &= 24,477 + j16,318 + 9,8058 \cdot \underline{-146,31^\circ} = \\ &= 24,477 + j16,318 - 8,15892 - j5,4393 = \\ &= 16,3178 + j10,8786 = 19,611 \cdot \underline{33,69^\circ} V \end{aligned}$$

$$U_{V1} = \sqrt{U_{V11}^2 + U_{V12}^2} = \sqrt{(12,5 \cdot \sqrt{2})^2 + 19,611^2} = 26,4 V$$

$$U_{V2} = \sqrt{U_{V21}^2 + U_{V22}^2} = \sqrt{(37,5 \cdot \sqrt{2})^2 + 29,417^2} = 60,64 V$$

12. Mreže izmjenične struje s feromagnetskim elementima

12.1. Mjerenjem gubitaka u feromagnetskom materijalu utvrđeno je da su gubici

$P_{Fe1} = 11,5 \text{ W/kg}$ pri frekvenciji $f_1 = 50 \text{ Hz}$, a pri frekvenciji $f_2 = 100 \text{ Hz}$ iznose

$P_{Fe2} = 9 \text{ W/kg}$. Koliki su pri tome gubici zbog vrtložnih struja, a koliki zbog histereze, ako je $U_1 = U_2 = U$?

$$P_v = \sigma_v \cdot f^2 \cdot B_m^2 \quad f_2 = 2 \cdot f_1$$

$$P_h = \sigma_h \cdot f \cdot B_m^2$$

$$U = 4,44 \cdot f \cdot N \cdot S \cdot B = k \cdot f \cdot B$$

$$k \cdot f_1 \cdot B_1 = k \cdot f_2 \cdot B_2 \quad \Rightarrow \quad B_2 = B_1 \cdot \frac{f_1}{f_2} = \frac{B_1}{2}$$

$$P_{Fe1} = P_{v1} + P_{h1} = \sigma_v \cdot f_1^2 \cdot B_{1m}^2 + \sigma_h \cdot f_1 \cdot B_{1m}^2$$

$$P_{Fe2} = P_{v2} + P_{h2} = \sigma_v \cdot f_2^2 \cdot B_{2m}^2 + \sigma_h \cdot f_2 \cdot B_{2m}^2 = \sigma_v \cdot (2f_1)^2 \cdot \left(\frac{B_{1m}}{2}\right)^2 + \sigma_h \cdot 2f_1 \cdot \frac{B_{1m}^2}{4}$$

$$P_{Fe2} = \sigma_v \cdot f_1^2 \cdot B_{1m}^2 + \frac{1}{2} \cdot \sigma_h \cdot f_1 \cdot B_{1m}^2 = P_{v1} + \frac{1}{2} \cdot P_{h1}$$

$$9 = P_{v1} + \frac{1}{2} \cdot P_{h1}$$

$$11,5 = P_{v1} + P_{h1} \quad \Rightarrow \quad P_{v1} = 11,5 - P_{h1}$$

$$9 = 11,5 - P_{h1} + \frac{1}{2} \cdot P_{h1}$$

$$\frac{P_{h1}}{2} = 2,5 \text{ / } 2$$

$$P_{h1} = 5 \text{ W/kg} \quad \Rightarrow \quad P_{v1} = 11,5 - 5 = 6,5 \text{ W/kg} = P_{v2}$$

$$P_{h2} = \frac{P_{h1}}{2} = 2,5 \text{ W/kg}$$

12.2. Svitak čiji je otpor $R = 2 \Omega$, namotan je na feromagnetsku jezgru i priključen na napon

220 V , frekvencije $f = 50 \text{ Hz}$. Struja svitka je $I = 2,5 \text{ A}$, a faktor snage $\cos \varphi = 0,1$.

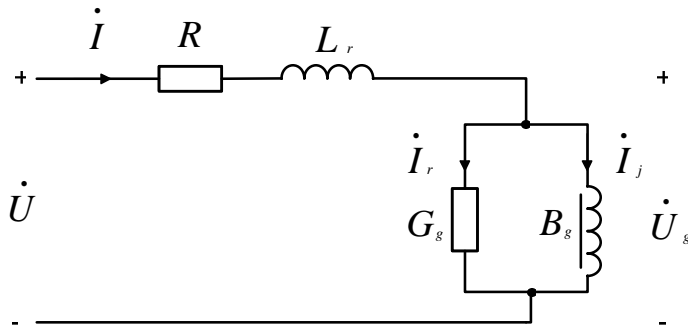
Treba odrediti gubitke u bakru i željezu. Rasipni tok je zanemariv.

$$P_{cu} = I^2 \cdot R = 2,5^2 \cdot 2 = 12,5 \text{ W}$$

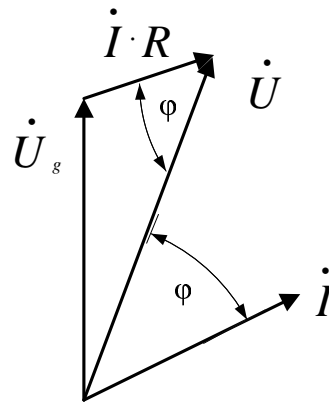
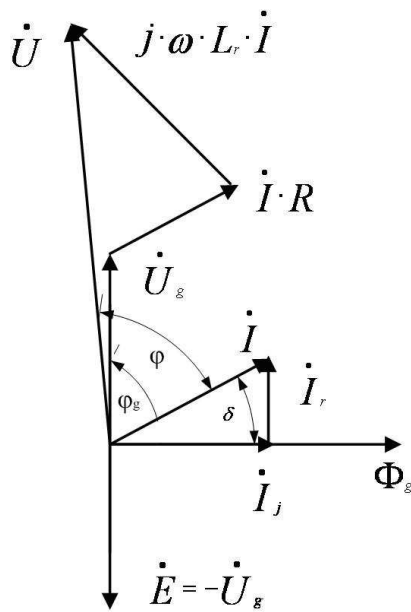
$$P = U \cdot I \cdot \cos \varphi = 220 \cdot 2,5 \cdot 0,1 = 55 \text{ W}$$

$$P_{Fe} = P - P_{cu} = 55 - 12,5 = 42,5 \text{ W}$$

12.3. U zadatku 12.2. treba nacrtati nadomjesnu shemu svitka i fazorski dijagram, te odrediti parametre nadomjesne sheme.



$$\begin{aligned} R &= 2 \, \Omega \\ U &= 220 \, V \\ f &= 50 \, Hz \\ I &= 2,5 \, A \\ \cos \varphi &= 0,1 \\ \varphi &= 84,261^\circ \\ P_{Fe} &= 42,5 \, W \\ L_r &= 0 \end{aligned}$$



jednostavnije:

$$\begin{aligned} U_g &= \sqrt{U^2 + (I \cdot R)^2 - 2 \cdot U \cdot I \cdot R \cdot \cos \varphi} = \\ &= \sqrt{220^2 + 5^2 - 2 \cdot 220 \cdot 5 \cdot 0,1} = 219,56 \, V \\ Y_g &= \frac{I}{U_g} = \frac{2,5}{219,56} = 0,0113866 \, S \end{aligned}$$

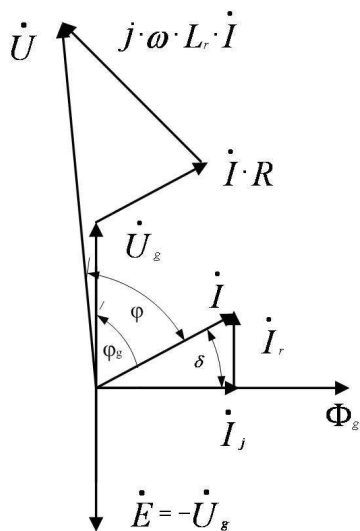
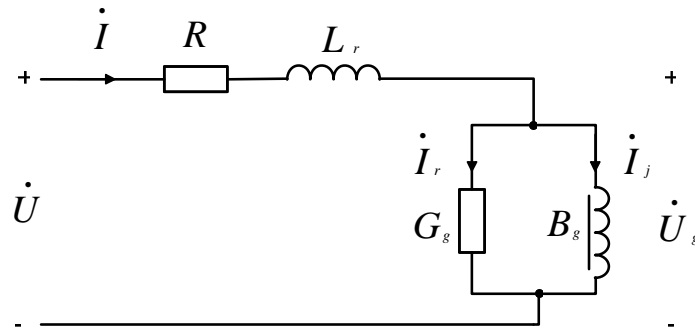
$$\begin{aligned} G_g &= \frac{P_{Fe}}{U_g^2} = \frac{42,5}{219,56^2} = 0,88165 \cdot 10^{-3} \, S \\ B_g &= \sqrt{Y_g^2 - G_g^2} = 11,352 \cdot 10^{-3} \, S \end{aligned}$$

12.4. U svitku sa željezom izmjereni su ukupni gubici $P_1 = 20 \text{ W}$ pri struji $I_1 = 2 \text{ A}$. Ako se izvadi jezgra, tada su pri struji $I_2 = 8 \text{ A}$ izmjereni gubici $P_2 = 64 \text{ W}$. Treba odrediti gubitke u bakru i željezu u prvom slučaju.

$$\begin{aligned} \text{a)} \quad P_1 &= 20 \text{ W} & P_{cu} &= I_1^2 \cdot R = 2^2 \cdot 1 = 4 \text{ W} \\ I_1 &= 2 \text{ A} & P_{Fe} &= P_1 - P_{cu} = 20 - 4 = 16 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad P_2 &= 64 \text{ W} = I_2^2 \cdot R \Rightarrow R \cdot \frac{P_2}{I_2^2} = \frac{64}{8^2} = 1 \Omega \\ I_2 &= 8 \text{ A} \end{aligned}$$

12.5. Svitak sa željezom priključen je na napon $U = 142 \text{ V}$ i $f = 50 \text{ Hz}$, pri čemu je izmjerena struja $I = 5,7 \text{ A}$ i radna snaga $P = 106 \text{ W}$. Treba odrediti gubitke i nacrtati nadomjesnu shemu i fazorski dijagram, ako je $R = 1 \Omega$ a rasipni tok $\Phi_r = 0,05 \Phi_g$.



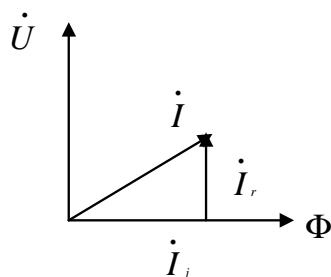
$$\begin{aligned} P_{cu} &= I^2 \cdot R = 5,7^2 \cdot 1 = 32,49 \text{ W} \\ P_{Fe} &= P - P_{cu} = 106 - 32,49 = 73,51 \text{ W} \\ \cos \varphi &= \frac{P}{U \cdot I} = \frac{106}{142,5,7} = 0,13096 \\ \varphi &= 82,4749^\circ \\ \dot{U}_g &= \dot{U} - \dot{I} \cdot R - j \cdot \dot{I} \cdot X_r \\ \dot{U}_g &= j \cdot U_g \\ \dot{U} &= U \angle \sigma + \varphi \\ \dot{I} &= I \angle \sigma \\ \Phi_r &= 0,05 \Phi_g \\ I \cdot X_r &= 0,05 U_g \end{aligned}$$

$$\begin{aligned}
U_g^2 &= U^2 + (I \cdot R)^2 + (I \cdot X_r)^2 - 2 \cdot U \cdot (I \cdot R) \cdot \cos \varphi - 2 \cdot U \cdot (I \cdot X_r) \cdot \sin \varphi \\
U_g^2 &= 142^2 + (5,7 \cdot 1)^2 + (0,05 \cdot U_g)^2 - 2 \cdot 142 \cdot 5,7 \cdot 1 \cdot \cos 82,475^\circ - 2 \cdot 142 \cdot 0,05 \cdot U_g \cdot \sin 82,475^\circ \\
U_g^2 &= 19984,49 + 0,0025 \cdot U_g^2 - 14,0777 \cdot U_g \\
0,9975 \cdot U_g^2 + 14,0777 \cdot U_g - 19984,49 &= 0 \\
U_{g1} &= 134,66 \text{ V} \\
U_{g2} &= -148,776 \text{ V} - \text{nije rješenje}
\end{aligned}$$

$$\begin{aligned}
G_g &= \frac{P_{Fe}}{U_g^2} = \frac{73,51}{134,66^2} = 4,054 \cdot 10^{-3} \text{ S} \\
Y_g &= \frac{I}{U_g} = \frac{5,7}{134,66} = 0,04233 \text{ S} \\
B_g &= \sqrt{Y_g^2 - G_g^2} = \sqrt{0,04233^2 - 0,004054^2} = 0,042133 \text{ S} \\
X_r &= \frac{U_r}{I} = \frac{0,05 \cdot U_g}{I} = \frac{0,05 \cdot 134,66}{5,7} = 1,18 \Omega \\
(12.9) \Rightarrow \sigma &= \arcsin \cdot \frac{P_{Fe}}{I \cdot U_g} = \arcsin \cdot \frac{73,51}{5,7 \cdot 134,66} = 5,5^\circ
\end{aligned}$$

$$\begin{aligned}
U_g &= U \cdot \sin(\varphi + \sigma) - I \cdot R \cdot \sin \sigma - I \cdot X_r \cdot \sin(\sigma + 90^\circ) = \\
&= 142 \cdot \sin(82,4749^\circ + 5,5^\circ) - 5,7 \cdot 1 \cdot \sin 5,5^\circ - 5,7 \cdot 1,18 \cdot \cos 5,5^\circ = \\
&= 141,911 - 0,54632 - 6,695 = 134,67 \text{ V}
\end{aligned}$$

12.6. Svitak sa željeznom jezgrom zanemarivog otpora i rasipnog toka priključen je na napon $U = 220 \text{ V}$ i $f = 50 \text{ Hz}$. Željezna jezgra presjeka $S = 50 \text{ cm}^2$, a srednja je duljina jezgre $l = 60 \text{ cm}$ koja ima zračni raspor $l_0 = 1 \text{ mm}$. Treba odrediti broj zavoja N i struju ako je $B = 1,4 \text{ T}$, pri čemu je jakost polja u željezu $H = 1410 \text{ A/m}$, a gubici u željezu $p_{1,4} = 7 \text{ W/kg}$.



$$\begin{aligned}
\omega &= 2 \cdot \pi \cdot f = 314 \text{ rad/s} \\
\Phi &= S \cdot B = 50 \cdot 10^{-4} = 7 \cdot 10^{-3} \text{ Vs} \\
U_m &= \omega \cdot N \cdot \Phi \\
N &= \frac{U \cdot \sqrt{2}}{\omega \cdot \Phi} = \frac{220 \cdot \sqrt{2}}{314 \cdot 7 \cdot 10^{-3}} = 141,5 \approx 142 \text{ zavoja} \\
V &= S \cdot l = 50 \cdot 10^{-4} \cdot 0,6 = 3 \cdot 10^{-3} \text{ m}^3 \\
m &= \rho \cdot V = 7,86 \cdot 10^3 \cdot 3 \cdot 10^{-3} = 23,58 \text{ kg} \\
P_{Fe} &= p_{1,4} \cdot m = 7 \cdot 23,58 = 165,06 \text{ W}
\end{aligned}$$

$$G_g = \frac{P_{Fe}}{U^2} = \frac{165,06}{220^2} = 0,00341 \text{ S}$$

$$I_r = U \cdot G_g = 220 \cdot 0,00341 = 0,7503 \text{ A}$$

$$H_0 = \frac{B}{1,1 \cdot \mu_0} = \frac{1,4}{1,1 \cdot 4 \cdot \pi \cdot 10^{-7}} = 1,0128 \cdot 10^6 \text{ A/m}$$

$$N \cdot I_{jm} = H \cdot l + H_0 \cdot l_0 = 1410 \cdot 0,6 + 1,0128 \cdot 10^6 \cdot 10^{-3} = 846 + 1012,8 = 1858,8 \text{ A}$$

$$I_{jm} = 13,09 \text{ A} \Rightarrow I_j = \frac{I_{jm}}{\sqrt{2}} = 9,256 \text{ A}$$

$$I = \sqrt{I_j^2 + I_r^2} = \sqrt{9,256^2 + 0,7503^2} = 9,286 \text{ A}$$

12.7. Paralelni spoj svitka sa željezom i kondenzatora kapaciteta $C = 15 \mu\text{F}$ priključen je na naponski izvor $e = 72 \sin(314t) \text{ V}$. Treba odrediti efektivnu vrijednost struje izvora ako je poznata ovisnost struje svitka i ulančanog toka, dana relacijom $i_L = 0,4\psi + 12\psi^3$.

$$\frac{d\psi}{dt} = U_m \sin \omega t \rightarrow \psi = -\frac{U_m}{\omega} \cos \omega t \quad \cos^3 \alpha = \frac{1}{4}(\cos 3\alpha + 3\cos \alpha)$$

$$\begin{aligned} i_L = 0,4\psi + 12\psi^3 &= -0,4 \frac{U_m}{\omega} \cos \omega t - 12 \frac{U_m^3}{\omega^3} \cos^3 \omega t = \\ &= -\frac{3}{314^3} U_m^3 \cos 3\omega t - \left(\frac{9}{314^3} U_m^3 + \frac{0,4}{314} U_m \right) \cos \omega t \end{aligned}$$

$$i_C = \omega C U_m \cos \omega t$$

$$i = i_L + i_C = -\frac{3}{314^3} U_m^3 \cos 3\omega t - \left(\frac{9}{314^3} U_m^3 + \frac{0,4}{314} U_m - 314 \cdot 15 \cdot 10^{-6} U_m \right) \cos \omega t$$

$$I_L = \sqrt{\frac{1}{2} \left(\frac{3U_m^3}{314^3} \right)^2 + \frac{1}{2} \left[\frac{U_m}{314} \left(9 \frac{U_m^2}{314^2} + 0,4 \right) \right]^2}$$

$$I_L = \frac{U_m}{314} \sqrt{45 \frac{U_m^4}{314^4} + 3,6 \frac{U_m^2}{314^2} + 0,08} \quad (1)$$

$$I_C = 3,34 \cdot 10^{-3} U_m$$

$$\begin{aligned} I &= \sqrt{\frac{1}{2} \left(\frac{3}{314^3} U_m^3 \right)^2 + \frac{1}{2} \left[\frac{U_m}{314} \left(\frac{9U_m^2}{314^2} - 1,07894 \right) \right]^2} \\ &= \frac{U_m}{314} \sqrt{\frac{1}{2} \left[\frac{9U_m^4}{314^4} + \frac{81U_m^4}{314^4} - \frac{2 \cdot 9 \cdot U_m^2}{314^2} \cdot 1,07894 + 1,07894^2 \right]} \end{aligned}$$

$$I = \frac{U_m}{314} \sqrt{45 \frac{U_m^4}{314^4} - 9,71 \frac{U_m^2}{314^2} + 0,582} \quad (2)$$

$$= \frac{72}{314} \sqrt{45 \frac{72^4}{314^4} - 9,71 \frac{72^2}{314^2} + 0,582} = 0,1015 \text{ A}$$

ferorezonancija struja:

tražimo ekstreme izraza (2): $\rightarrow \frac{\partial I}{\partial U_m} = 0 = \frac{45 \cdot 6 \frac{U_m^5}{314^6} - 9,71 \cdot 4 \frac{U_m^3}{314^4} + 0,582 \cdot 2 \frac{U_m}{314^2}}{\sqrt{45 \frac{U_m^6}{314^6} - 9,71 \frac{U_m^4}{314^4} + 0,582 \frac{U_m^2}{314^2}}}$

$$\frac{U_m}{314^2} \left(45 \cdot 6 \frac{U_m^4}{314^4} - 9,71 \cdot 4 \frac{U_m^2}{314^2} + 0,582 \cdot 2 \right) = 0$$

$$U_{m1} = 99,98 \text{ V}$$

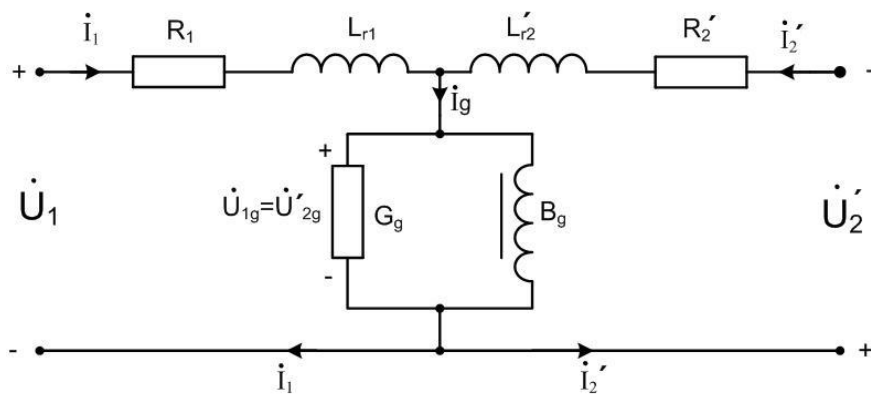
-ferorezonancija

$$\frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x}}$$

$$U_{m2} = 64,8 \text{ V}$$

-prvi maksimum struje

- 12.8. Jednofazni transformator nominalne snage $S_1 = 3,5 \text{ MVA}$ i nominalnih napona $U_1 = 35 \text{ kV}$ i $U_2 = 380 \text{ kV}$ ima gubitke u praznom hodu $P_{1ph} = 20 \text{ kW}$ uz nominalni napon U_1 , pri čemu je struja praznog hoda 5 % nominalne. Napon kratkog spoja je 10 % nominalnog, pri čemu je struja primara nominalna, a snaga kratkog spoja je $P_{1k} = 50 \text{ kW}$. Nacrtajte fazorski dijagram uz pretpostavku da je $R_1 = R_2'$, $X_{r1} = X_{r2}'$ i odredite parametre nadomjesne sheme.



$$I_1 = I_{1K} = \frac{S_1}{U_1} = \frac{3,5 \cdot 10^6}{35 \cdot 10^3} = 100 \text{ A}$$

$$U_{1K} = 0,1 \cdot 35 \cdot 10^3 = 3500 \text{ V}$$

$$Z_{1K} = \frac{U_{1K}}{I_{1K}} = \frac{3500}{100} = 35 \Omega$$

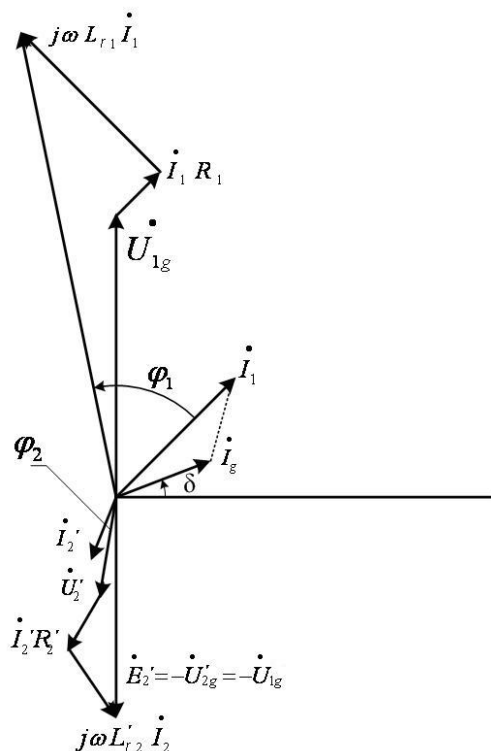
$$\cos \varphi_K = \frac{P_{1K}}{U_{1K} \cdot I_{1K}} = \frac{50 \cdot 10^3}{3500 \cdot 100} = 0,142847$$

$$\varphi_K = 81,786^\circ$$

$$\dot{Z}_{1K} = 35 \angle 81,786^\circ = 5 + j 34,641 \Omega$$

$$R_1 = R_2' = 2,5 \Omega$$

$$X_1 = X_2' = 17,32 \Omega$$



$$Z_{1PH} = \frac{U_{1PH}}{I_{1PH}} = \frac{35 \cdot 10^3}{0,05 \cdot 100} = 7000 \Omega$$

$$\cos \varphi_{PH} = \frac{P_{1PH}}{U_{1PH} \cdot I_{1PH}} = \frac{20 \cdot 10^3}{35 \cdot 10^3 \cdot 5} = 0,114286$$

$$\varphi_{PH} = 83,4375^\circ$$

$$\dot{Z}_{1PH} = 7000 \angle 83,4375^\circ \Omega$$

$$\dot{Z}_{1PH} \approx \dot{Z}_g$$

$$\dot{Y}_g = \frac{1}{\dot{Z}_g} = \frac{1}{7000 \angle 83,4375^\circ} =$$

$$= 1,633 \cdot 10^{-5} - j1,42 \cdot 10^{-4} S = G_g - jB_g$$

12.9. Jednofazni transformator nominalne snage $S_l = 20$ kVA, napona $U_l = 5$ kV, frekvencije $f = 50$ Hz i prienosnog omjera $N_1/N_2 = 20$ ima napon kratkog spoja $U_{lk} = 300$ V, pri čemu je snaga kratkog spoja $P_{lk} = 960$ W pri nominalnoj struji. Treba odrediti: a) nominalni napon sekundara, te primarnu struju; b) impedanciju kratkog spoja i postotni iznos napona kratkog spoja; c) rasipne induktivitete; d) struju primara pri kratkom spoju sekundara i nominalnom naponu primara. Pretpostaviti da je $R_l = R_2'$, $X_{r1} = X_{r2}'$.

$$a) \quad I_{ln} = \frac{S_l}{U_l} = \frac{20 \cdot 10^3}{5 \cdot 10^3} = 4 A = I_{lk}$$

$$U_{2n} = \frac{U_l}{\frac{N_1}{N_2}} = \frac{5 \cdot 10^3}{20} = 250 V$$

$$\omega = 2\pi f = 314 \text{ rad/s}$$

$$b) \quad Z_{lk} = \frac{U_{lk}}{I_{lk}} = \frac{300}{4} = 75 \Omega$$

$$\cos \varphi_k = \frac{P_{lk}}{U_{lk} \cdot I_{lk}} = \frac{960}{300 \cdot 4} = 0,8$$

$$\varphi_k = 36,87^\circ$$

$$\dot{Z}_k = 75 \angle 36,87^\circ = 60 + j45 \Omega$$

$$u_k(\%) = \frac{U_k}{U_{ln}} \cdot 100\% = \frac{300}{5 \cdot 10^3} \cdot 100\% = 6\%$$

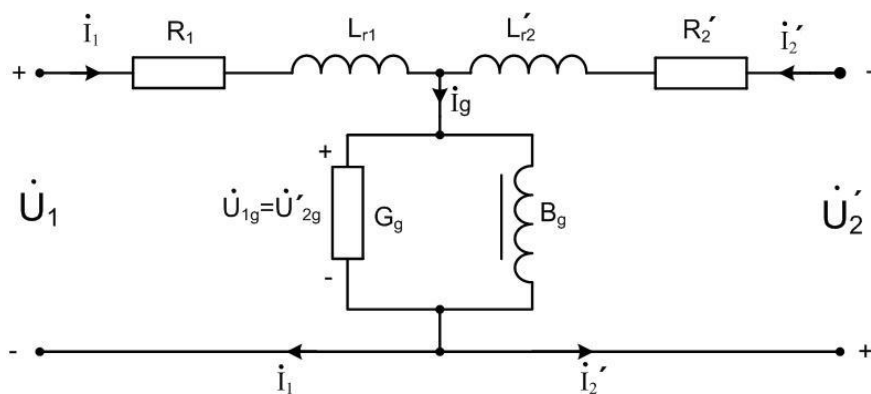
$$c) \quad X_{r1} = X_{r2}' = \frac{X_k}{2} = 22,5 \Omega = \omega L_{r1}$$

$$L_{r1} = \frac{22,5}{314} = 71,62 \text{ mH} = L_{r2}'$$

$$L_{r2} = \frac{L_{r2}'}{\left(\frac{N_1}{N_2}\right)^2} = \frac{71,62 \cdot 10^{-3}}{20^2} = 0,179 \text{ mH}$$

$$d) \quad I_{K1} = \frac{U_{1n}}{Z_{1K}} = \frac{5 \cdot 10^3}{75} = 66,67 \text{ A}$$

12.10. Jednofazni transformator nominalnih napona $U_1 = 10000 \text{ V}$ i $U_2 = 500 \text{ V}$ ima omjer gubitaka $P_{Cu}/P_{Fe} = 1,5$, opterećen je trošilom snage $S_2 = 100 \text{ kVA}$ i $\cos \varphi_2 = 1$, pri čemu je djelotvornost $\eta = 0,95$. Odredite otpore svitaka ako je prijenosni omjer $n = U_1/U_2 = 20$, a $R_1 = R_2'$, $X_{r1} = X_{r2}'$.



$$S_2 = U_2 I_2 \cos \varphi_2 = 500 \cdot I_2 \cdot 1 = 100 \cdot 10^3 = P_2$$

$$I_2 = 200 \text{ A}$$

$$I_2' = \frac{I_2}{n} = \frac{200}{20} = 10 \text{ A}$$

$$P_1 = \frac{P_2}{\eta} = \frac{100 \cdot 10^3}{0,95} = 105263 \text{ W}$$

$$P_{Cu} = 1,5 P_{Fe}$$

$$P_g = P_1 - P_2 = 5263,16 \text{ W} = P_{Cu} + P_{Fe} = 1,5 P_{Fe} + P_{Fe} = 2,5 P_{Fe}$$

$$P_{Fe} = \frac{5263,16}{2,5} = 2105,26 \text{ W}$$

$$P_{Cu} = 3157,89 \text{ W}$$

$$P_{R1} = \frac{P_{Cu}}{2} = 1578,95 \text{ W} = I_1^2 R_1$$

$$R_1 = \frac{P_{R1}}{I_1^2} = \frac{1578,95}{10^2} = 15,79 \Omega = R_2'$$

$$R_2 = \frac{R_2'}{n^2} = \frac{15,79}{20^2} = 39,47 \text{ m}\Omega$$

12.11. Jednofazni transformator nominalne snage 100 kVA i napona $1000/230 \text{ V}$; frekvencije $f = 50 \text{ Hz}$ ima $u_{1k} (\%) = 5,5 \%$. Ako su gubici kratkog spoja $P_{1k} = 800 \text{ W}$, treba odrediti impedanciju kratkog spoja \dot{Z}_k , uz pretpostavku da je $R_1 = R_2'$, $X_{r1} = X_{r2}'$.

$$U_{1k} = u_{1k} \cdot U_{1N} = 0,055 \cdot 1000 = 55 \text{ V}$$

$$I_{1N} = \frac{S_1}{U_1} = \frac{100 \cdot 10^3}{10^3} = 100 \text{ A} = I_{1k}$$

$$Z_k = \frac{U_{1k}}{I_{1k}} = \frac{55}{100} = 0,55 \Omega$$

$$\cos \varphi_k = \frac{P_{1k}}{U_{1k} \cdot I_{1k}} = \frac{800}{55 \cdot 100} = 0,14545$$

$$\varphi_k = 81,636^\circ$$

$$\dot{Z}_k = 0,55 \angle 81,636^\circ \Omega$$

12.12. Jednofazni transformator nominalne snage $S_I = 50 \text{ kVA}$ i $n = 2300/230 \text{ V}$ ima $R_I = 0,5 \, \Omega$ i $R_2 = 0,005 \, \Omega$, te gubitke u željezu $P_{Fe} = 290 \text{ W}$. Treba izračunati stupanj djelovanja za opterećenje: a) 50 kW , $\cos \varphi_2 = 1$; b) 5 kW , $\cos \varphi_2 = 1$; c) 50 kVA , $\cos \varphi_2 = 0,8$.

a)

$$P_2 = 50 \text{ kW} = U_2 I_2 \cos \varphi_2 = 230 \cdot I_2 \cdot 1 \rightarrow I_2 = \frac{50 \cdot 10^3}{230} = 217,39 \text{ A}$$

$$\cos \varphi_2 = 1$$

$$I_1 = I_1' = \frac{I_2}{n} = \frac{217,39}{10} = 21,739 \text{ A}$$

$$P_{Cu} = I_1^2 R_1 + I_2^2 R_2 = 21,739^2 \cdot 0,5 + 217,39^2 \cdot 0,005 = 472,59 \text{ W}$$

$$\eta = \frac{P_2}{P_1} = \frac{50 \cdot 10^3}{50 \cdot 10^3 + 472,59 + 290} = 98,8\%$$

b)

$$P_2 = 5 \text{ kW} = I_2 U_2 \cdot 1 \rightarrow I_2 = \frac{5 \cdot 10^3}{230} = 21,739 \text{ A}$$

$$\cos \varphi_2 = 1$$

$$I_1 = \frac{I_2}{n} = \frac{21,739}{10} = 2,1739 \text{ A}$$

$$P_{Cu} = 2,1739^2 \cdot 0,5 + 21,739^2 \cdot 0,005 = 4,726 \text{ W}$$

$$\eta = \frac{P_2}{P_1} = \frac{5 \cdot 10^3}{5 \cdot 10^3 + 4,726 + 290} = 94,93 \%$$

c)

$$S = 50 \text{ kVA} = U_2 I_2 \rightarrow I_2 = \frac{50 \cdot 10^3}{230} = 217,39 \text{ A} \rightarrow I_2 = 21,739 \text{ A}$$

$$\cos \varphi_2 = 0,8$$

$$P_{Cu} = I_1^2 R_1 + I_2^2 R_2 = 21,739^2 \cdot 0,5 + 217,39^2 \cdot 0,005 = 472,59 \text{ W}$$

$$P_2 = 50 \cdot 10^3 \cdot 0,8 = 40 \cdot 10^3 \text{ W}$$

$$\eta = \frac{P_2}{P_2 + P_g} = \frac{40 \cdot 10^3}{40 \cdot 10^3 + 472,59 + 290} = 98,13 \%$$

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